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Problem 3 Mean Value

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Let

$$b(x) = (x-a)^2,\ 0 \le x \le 1,\ 0 \le a \le 1.$$

Denote the mean value of the function $b(x)$ on the interval from s to t by the formula

$$M(s,t) = \frac{1}{t-s}\int_s^t b(x)dx,\ \ 0 \le s \le 1,\ \ 0 \le t \le 1.$$

Prove that

$$M(s,t) \ge \frac{b(t)}{4} \ \text{ for all } \ 0 \le s \le 1,\ \ 0 \le t \le 1.$$