University of Utah, Department of Mathematics
Algebra 2 Qualifying Exam
May 2021

There are five problems on this exam. You may attempt as many problems as you wish; two correct solutions count as a *pass*, and three correct as a *high pass*. Show all of your work and provide reasonable justification for your answers.

1. Determine whether the following two fields K and L are isomorphic; if they are isomorphic then construct an explicit isomorphism between them.

$$K = \mathbb{Q}[x]/(x^2+2x+2), \;\; L = \mathbb{Q}(i) \subset \mathbb{C}.$$

2. Suppose $f(x) \in \mathbb{Q}[x]$ is a polynomial with Galois group of order $594 = 2 \cdot 3^3 \cdot 11$. Show that f is solvable in radicals.

3. Recall that the *standard representation* of S_n is the subspace

$$V_n := \{(x_1, \ldots, x_n) \in \mathbb{C}^n, x_1 + x_2 + \ldots + x_n = 0\}$$

with S_n acting by permutation of the coordinates. Show that $\wedge^2 V_5$ is an irreducible representation of S_5.

4. Show that if L/K is Galois and $f \in K[x]$ is monic irreducible, then every irreducible factor of f in $L[x]$ has the same degree.

5. Suppose σ and τ are two elements of A_5 such that, for every finite dimensional complex representation ρ of S_5, $\chi_\rho(\sigma) = \chi_\rho(\tau)$ (here χ_ρ denotes the character of ρ). Are σ and τ conjugate in A_5?