Practice Problems for 5010 Final

1. In how many ways can 2 novels, 3 mathematics books, and 1 history book be arranged on a bookshelf if

   (a) the books can be arranged in any order;

   (b) the mathematics books must be together and the novels must be together;

   (c) the novels must be together but the other books can be arranged in any order?

2. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.

   (a) What percentage of males smoke neither cigars nor cigarettes?

   (b) What percentage smoke cigarettes but not cigars?

3. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent as Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is

   (a) an Independent;

   (b) a Liberal;

   (c) a Conservative?

   (d) What fraction of voters participated in the local election?

4. A particular stock is currently traded at the price s. In this binomial model, we assume that after each time period the price is multiplied by either $u > 1$, with probability p, or $d = 1/u$, with probability $1 - p$. We assume that successive stock price movements are independent from each other. Let X be the price after three time periods.

   (a) List all the possible values of X in this model (in terms of s) and compute its probability mass function;

   (b) Compute $E[\max(X-s, 0)]$ (which refers to the expected payoff of a call option).

5. In Problem No. 4, if we consider 100 time periods. Approximate the probability that the stock price will be up at least 10 percent after the next 100 time periods. What assumptions do you need in order to make this approximation?

6. The joint probability density function of X and Y is given by

$$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) \qquad 0 < x < 1,\ 0 < y < 2$$

   (a) Verify that this is indeed a joint density function.

   (b) Find $P\{X < Y\}$.

   (c) Find the marginal density function $f_X(x)$.

   (d) Find $E[X]$.

   (e) Find $E[X^2 \,|\, Y = 1]$.

7. The time (in hours) required to repair a car is an exponentially distributed random variable with parameter $\lambda = 1/2$. What is

   (a) the probability that a repair time exceeds 2 hours;

   (b) the conditional probability that a repair takes at least 10 hours, given that it has been worked on for over 5 hours.

8. Let X be the number of 3's and Y the number of 2's that occur in n rolls of a fair die.

   (a) Find $P\{X = i, Y = j\}$ for $0 \le i, j \le n$.

   (b) Compute $\mathrm{Cov}(X, Y)$.

9. There are three configurations A, B, C for a particular laptop model that are being sold by an online retailer. Each laptop sold will be of a particular configuration with probability 1/3 and that is independent of previous sales. We assume the supplies are unlimited.

   (a) Compute the mean number of the laptops sold before the first configuration A is sold.

   (b) Compute the mean number of different configurations that are sold before the first configuration A is sold.

10. An insurance company has 10,000 automobile policyholders. The expected yearly claim per policyholder is \$240 with a standard deviation of \$800. Approximate the probability that the total yearly claim exceeds \$2.5 million.