Rows & Columns

In this chapter we'll learn how to multiply a row of n numbers and a column of n numbers to obtain a single number. Actually, in this class, n will always be either 2 or 3, though it could be any natural number.

Rows

A *row* of 2 numbers is just two numbers written left-to-right. For example, $(3, 4)$ and $(-2, 0)$ are each a row of 2 numbers.

The *entries* of a row are the numbers that make up the row. For example, the first entry of the row $(3, 4)$ is 3. The second entry of $(3, 4)$ is 4.

An example of a row of three numbers is $(2, -4, 6)$. The first entry of this row is 2, the second entry is -4, and the third entry is 6.

Columns

A *column* of 2 numbers is two numbers written top-to-bottom. For example,

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

is a column of two numbers. Its first entry is 4 and its second entry is 1.

(Caution: We write a column of 2 numbers exactly as we wrote binomial coefficients earlier in the semester. Although they are written in the same way, they mean different things. We just have to use the context of the problem we are working on to interpret the correct meaning. This is similar to how *there* and *they're* sound the same, but it's always clear in conversation which of these two words a person is using.)

As another example of a column of numbers, we write the column of 3 numbers with entries 5, 0, and -1 as

$$\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$$

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Multiplying rows (on the left) and columns (on the right)

To multiply a row of numbers and a column of numbers, the row and the column must have the same number of entries. For example, we can multiply a row of 3 numbers and a column of 3 numbers, but we cannot multiply a row of 3 numbers and a column of 2 numbers.

Also, to multiply a row of numbers and a column of numbers, the row must be written on the left, and the column must be written on the right.

To perform the multiplication of the row and the column, multiply the first entry in the row and the first entry of the column. That's the first product.

Now take the product of the second entry in the row and the second entry in the column. That's the second product.

If the row and column being multiplied each have 3 entries, then there will also be a third product obtained from multiplying the third entries of the row and column.

Now sum the products: the first, and the second, and the third (if the row and column each have three entries). That's the answer.

Examples.

$$(3,4)\begin{pmatrix}1\\2\end{pmatrix} = 3\cdot 1 + 4\cdot 2 = 3+8 = 11$$

$$(-2,0)\begin{pmatrix}1\\-3\end{pmatrix} = -2\cdot 1 + 0\cdot(-3) = -2$$

More generally, the pattern for multiplying a row of two numbers and a column of two numbers is given by

$$(r_1,r_2)\begin{pmatrix}c_1\\c_2\end{pmatrix} = r_1c_1 + r_2c_2$$

Similarly, the pattern for multiplying a row of three number and a column of three numbers is given by

$$(r_1,r_2,r_3)\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = r_1c_1 + r_2c_2 + r_3c_3$$

For example,

$$(1, 2, -6)\begin{pmatrix}-2\\0\\4\end{pmatrix} = 1 \cdot (-2) + 2 \cdot 0 + (-6) \cdot 4 = -2 + 0 - 24 = -26$$

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Exercises

Find the following products of rows and columns.

1.) $(2, 8)\begin{pmatrix}1\\1\end{pmatrix}$

2.) $(-1, 0)\begin{pmatrix}0\\15\end{pmatrix}$

3.) $(2, 3)\begin{pmatrix}8\\-2\end{pmatrix}$

4.) $(3, -2)\begin{pmatrix}-2\\-4\end{pmatrix}$

5.) $(1, 0, 1)\begin{pmatrix}3\\4\\1\end{pmatrix}$

6.) $(-2, 2, 1)\begin{pmatrix}-2\\3\\2\end{pmatrix}$

7.) $(4, 6, -3)\begin{pmatrix}-2\\4\\-2\end{pmatrix}$

8.) $$(-6,-11,-13)\begin{pmatrix}5\\0\\0\end{pmatrix}$$

For #9-11, determine whether the given sequences are arithmetic, geometric, or neither.

9.) $-2, 6, -18, 54, \ldots$

10.) $-4, 1, 6, 11, \ldots$

11.) $-3, 2, -2, 5, \ldots$

For #12-15, find the 23rd term of the given sequence.

12.) $4, 7, 10, 13, \ldots$

13.) $4, 8, 16, 32, \ldots$

14.) $13, 23, 33, 43, \ldots$

15.) $3, 30, 300, 3000, \ldots$