Math 1320-6 Lab 5

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Name: ______________________________

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Instructions and due date:

- **Due:** 3 March 2016 at the start of class.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).

1. Give an example of a power series with the following radii of convergence: (a) $R = 0$; (b) $R = 3$; (c) $R = \infty$. Justify your answer.

   (a) $R = 0$

   (b) $R = 3$

   (c) $R = \infty$

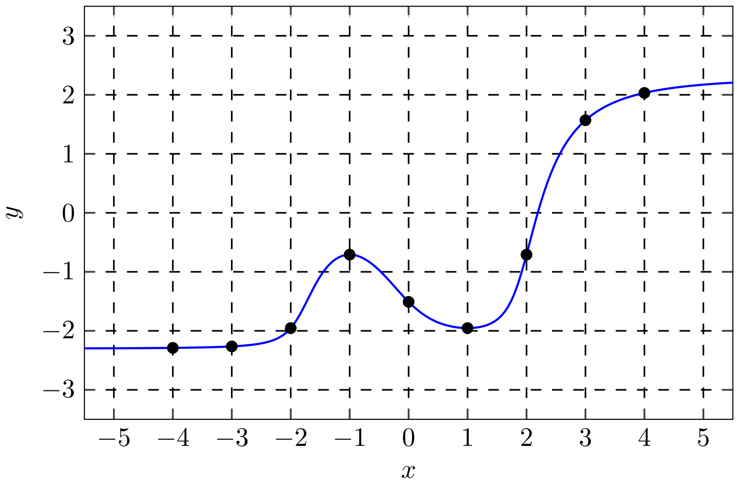
2. Consider the function $f(t) = (\arctan t)/t$.

   (a) Compute a power series expansion of $\mathcal{I}(x) = \int_0^x f(t)dt$. Write your answer in the form $\sum_{k=1}^{\infty} a_k x^{2k-1}$ for an appropriate choice of a_k. Compute the radius of convergence of the resulting power series expansion.

   (b) Let $\mathcal{I}_n(x) = \sum_{k=1}^{n} a_k x^{2k-1}$ be the n-th partial sum of the series in part (a) (i.e. it consists of the first n terms of the power series). Define E_n to be the error of approximating $\mathcal{I}(1)$ by $\mathcal{I}_n(1)$: $E_n = |\mathcal{I}(1) - \mathcal{I}_n(1)|$.

   What is the minimum number of terms N required so that $E_N < 0.01$? What if we require $E_N < 0.001$? $E_N < 0.000001 = 10^{-6}$? (Hint: Use the Alternating Series Estimation Theorem. Also: All three of your answers should be positive integers.)

3. Consider the following graph (in blue) of a function $f(x)$. Answer the following questions, and justify your answer with a short sentence or two.



(a) Is $\frac{3}{4} - x + \frac{1}{2}x^2 + \frac{1}{2}x^3 - \frac{5}{8}x^4 + \cdots$ the Maclaurin series of $f(x)$?

(b) Is $-\frac{5}{7} + \frac{5}{9}(x+1) + \frac{5}{4}(x+1)^2 + \frac{5}{12}(x+1)^3 - \frac{1}{7}(x+1)^4 + \cdots$ the Taylor series of $f(x)$ at the point $x = -1$?

(c) Is $\frac{8}{5} + \frac{8}{9}(x-3) + \frac{3}{18}(x-3)^2 + \frac{15}{31}(x-3)^3 - \frac{3}{11}(x-3)^4 + \cdots$ the Taylor series of $f(x)$ at the point $x = 3$?

4. The following limits represent some derivative of $f(x)$. Use Taylor series to determine which derivative it is.

   (a) $\lim_{h\to 0} \frac{1}{h^2}\left[f(x+h)+f(x-h)-2f(x)\right]$

   (b) $\lim_{h\to 0} \frac{1}{2h}\left[f(x+h)-f(x-h)\right]$