Math 1320-6 Lab 4

Name: _____

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Instructions and due date:

- **Due:** 11 February 2016 at the start of class.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).

1. Use the method of separation of variables to solve the following problems:

   (a) Let $y' = \dfrac{dy}{dx}$, and solve for y:

   $$(y + xy)y' = 1.$$

   (b) Find an equation for the curve that passes through $(0, 1)$ and whose slope at (x, y) is $6x(y - 1)^{2/3}$.

2. A glass of hot water is cooling down with surrounding temperature of 72 °F. The rate of change of the water temperature $T(t)$ is directly proportional to the difference between $T(t)$ and the surrounding temperature. Suppose that at $t = 0$ min, the water temperature is 100 °F, and drops to 82 °F after 10 min.

   (a) Set up the differential equation describing $T(t)$.

   (b) Solve the differential equation from part (a). (Hint: Your answer of $T(t)$ should have two unknowns: the constant of proportionality, and the arbitrary constant of integration (C).)

   (c) Use the two conditions given by the problem to solve the two unknowns from part (b).

3. A population $P(t)$ has constant relative birth and death rates α and β, respectively, and a constant emigration rate m (α, β and m are positive constants). Assume $\alpha > \beta$. The rate of change of the population at time t is modeled by

$$\frac{dP}{dt} = kP - m, \quad \text{where} \quad k = \alpha - \beta.$$

(a) Find the solution of this equation that satisfies the initial condition $P(0) = P_0$.

(b) What condition on m relative to kP_0 will lead to exponential expansion of the population? What condition on m relative to kP_0 will result in constant population? Population decline?

(c) In 1847, the population of Ireland was approximately 8 000 000, while the difference between the relative birth and death rates was 1.6 %. As a result of the potato famine in the 1840s and 1850s, 210 000 inhabitants per year emigrated from Ireland. Was the population expanding or declining?

4. Determine whether the sequence is convergent or divergent and explain why.

(a) $a_n = e^{\frac{1}{n}}$

(b) $a_n = (-1)^n \dfrac{n}{n^2+1}$

(c) $a_n = \dfrac{\sin(n\pi)}{n}$

(d) $a_n = \ln(n) - \ln(n-1)$

(e) $a_n = \sqrt{n} - \sqrt{n-1}$