Math 1320 - Lab 1

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Instructions:

- Draw sketches to improve your understanding of each exercise!
- For full credit, simplify your final answers.
- Work together! However your work should be your own (not copied from a group member).

Name: _____ Student ID: _____

1. Find the volume of the solid of revolution formed by rotating the finite region bounded by the graphs of $y = x$ and $y = x^4$ around the following two lines.

   (a) The x-axis

   (b) The line $x = -1$

2. Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = (x-1)^2$ and $x = y^2 + 1$ around the line $y = -1$.

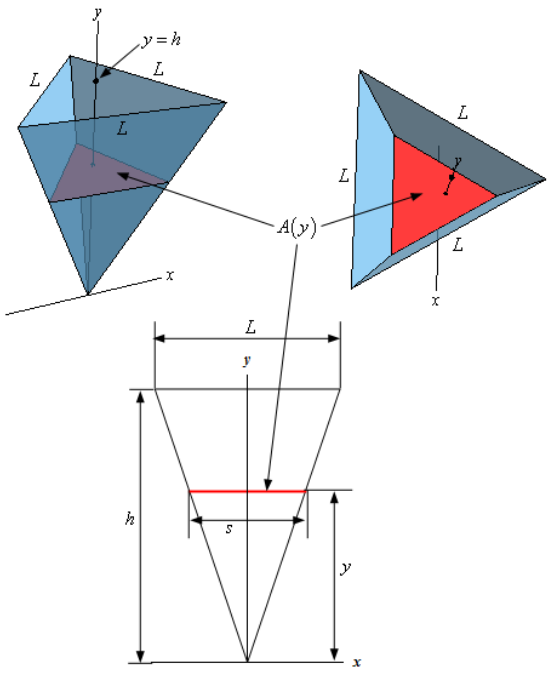
3. A machinist has a hemisphere made out of some alloy metal, with radius 1 cm. They wish to have a volume of exactly 2 cm^3 of this metal. What size of cylindrical hole should they bore through the center of the hemisphere to achieve this volume? That is: What should the radius of the cylindrical hole be?

4. In this exercise, we will find the volume of a pyramid of height h whose base is an equilateral triangle with side length L.

   (a) As in lecture, we can compute the volume of this object as follows:

$$V = \int_0^h A(y)\,\mathrm{d}y,$$

   where $A(y)$ is the cross sectional area (in red) in the following figure:



   Thus, we must determine $A(y)$. As a first step, consider the cross section shown with width s. What is the area of this cross section in terms of s? (*Hint: It is also an equilateral triangle.*)

(b) Now that we have the area for a particular width s, relate s and y to L and h. (*Hint: Use similar triangles.*)

(c) From this, obtain an explicit expression for $A(y)$, involving only L, h, and y, but not s.

(d) Compute the volume of the pyramid.