

Name: _____

Solutions

Math 4400 Quiz 3

June 9, 2016

Instructions: You have until the end of class to complete this quiz. This quiz is two pages, and worth 20 points. Make sure to write your name at the top of the quiz. Show all of your work for full credit!

1. (10 points) Let $a \in \mathbb{Z}$. Show that $a^2 - 2$ is not divisible by 5.

~~QED~~ $a^2 - 2$ is divisible by 5

$$\begin{array}{c} \updownarrow \\ a^2 - 2 \equiv 0 \pmod{5} \end{array}$$

Check :

$$0^2 - 2 \equiv 3 \pmod{5}$$

$$1^2 - 2 \equiv -1 \equiv 4 \pmod{5}$$

$$2^2 - 2 \equiv 2 \pmod{5}$$

$$3^2 - 2 = 7 \equiv 2 \pmod{5}$$

$$4^2 - 2 = 14 \equiv 4 \pmod{5}$$

Thus $a^2 - 2$ is not congruent to 0 mod 5
for any $a \in \mathbb{Z}$.

2. (10 points) Use the Euclidean Algorithm to find all incongruent solutions to $15x \equiv 12 \pmod{21}$.

Euclidean algo:

$$21 = 1 \cdot 15 + 6$$

$$15 = 2 \cdot 6 + 3$$

$$6 = 2 \cdot 3$$

$$\left. \begin{array}{l} 21 = 1 \cdot 15 + 6 \\ 15 = 2 \cdot 6 + 3 \\ 6 = 2 \cdot 3 \end{array} \right\} \rightarrow \begin{array}{l} 15 - 2 \cdot 6 = 3, \\ 15 - 2(21 - 15) = 3 \\ \Rightarrow 3 \cdot 15 - 2 \cdot 21 = 3 \end{array}$$

Thus $15 \cdot 3 \equiv 3 \pmod{21}$,

so $15 \cdot 12 \equiv 12 \pmod{21}$. (multiply each side by 4).

First solution: $x_0 = 12$.

$$\gcd(15, 21) = 3, \quad \frac{21}{\gcd(15, 21)} = 7.$$

So the three solutions are

$$x \equiv 12, x \equiv 12 + 7, x \equiv 12 + 14 \pmod{21}$$

I.e. 5, 12, and 19.