Name: Solutions

Math 4400 Quiz 1

May 24, 2017

Instructions: You have until the end of class to complete this quiz. This quiz is two pages, and worth 20 points. Make sure to write your name at the top of the quiz. Show all of your work for full credit!

1. (10 points) Let $a, b \in \mathbb{Z}$ with $a, b \neq 0$. Prove that $\gcd(a, b) = \gcd(a, b - a)$.

By definition, $\gcd(a,b)$ = maximal element of the set $S = \cancel{\{c\in} \{c \in \mathbb{Z} \mid c|a \text{ and } c|b\}$

$\gcd(a, b-a)$ = max'l element of $T = \{c \mid c|a \text{ and } c|b-a\}$, $c \in \mathbb{Z}$

So it's enough to show $S = T$.

Suppose $x \in S$. Then $\exists\, d, e \in \mathbb{Z}$, $dx = a$, $ex = b$.

Then $(e-d)x = b-a$, so $x | b-a$, so $x \in T$.

$\Rightarrow S \subseteq T$

Suppose $y \in T$. Then $\exists\, f, g \in \mathbb{Z}$ s.t. $fy = a$, $gy = b-a$. Then $(f+g)y = b$, so $y | b$.

$\Rightarrow T \subseteq S$. $\Rightarrow S = T$ ∎

2. (10 points) Prove that $\sqrt{6} = [2; \overline{2, 4}]$

To show $\sqrt{6} = 2 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{4 + \dots}}}}$, it's

enough to show $\sqrt{6} - 2 = \cfrac{1}{2 + \cfrac{1}{4 + \sqrt{6} - 2}}$

So we check:

$$\frac{1}{2 + \frac{1}{4 + \sqrt{6} - 2}} = \frac{1}{2 + \frac{1}{2 + \sqrt{6}}} = \frac{1}{\frac{4 + 2\sqrt{6} + 1}{2 + \sqrt{6}}}$$

$$= \frac{2 + \sqrt{6}}{5 + 2\sqrt{6}}$$

$$\sqrt{6} - 2 = \frac{2 + \sqrt{6}}{5 + 2\sqrt{6}} \iff (\sqrt{6} - 2)(5 + 2\sqrt{6}) = 2 + \sqrt{6}$$

$$\iff 5\sqrt{6} - 10 + 12 - 4\sqrt{6} = 2 + \sqrt{6}$$

$$\iff 2 + \sqrt{6} = 2 + \sqrt{6} \quad \checkmark \quad \blacksquare$$