Is the series $\sum a_n$ positive? ($a_n \geq 0$)

— yes → (If $\sum a_n$ converges, it converges absolutely.)

— no → **Consider $\sum |a_n|$. $\sum |a_n|$ is positive. Does $\sum |a_n|$ converge?**
  - yes → converges absolutely
  - no ↓

Find $\lim\limits_{n \to \infty} a_n$. Is $\lim\limits_{n \to \infty} = 0$?
- yes ↓
- no → diverges

Is $\sum a_n = \sum (-1) b_n$?
- yes → **Try Alternating Series Test: Is $\lim\limits_{n \to \infty} b_n = 0$ <u>AND</u> $b_n > b_{n+1}$?**
  - no → diverges
  - yes → converges conditionally
- no → Look at sequence of partial sums.

Is it a p-series? $\sum \frac{1}{n^p}$
- yes → $p > 1$ converges, $p \leq 1$ diverges
- no ↓

Is it geometric? $\sum r^n$
- yes → $|r| < 1$ converges to $\frac{1}{1-r}$, $|r| \geq 1$ diverges
- no ↓

Does a_n involve factorials or n as a power?
- yes → **Try Ratio Test: Calculate $\rho = \lim\limits_{n \to \infty} \frac{a_{n+1}}{a_n}$**
  - $\rho < 1$ → converges
  - $\rho > 1$ → diverges
  - $\rho = 1$ or ρ doesn't exist ↓
- no ↓

Does a_n involve only constant powers of n?
- yes → **Let b_n = expression of only highest terms.**
- no ↓

Does a_n involve $\sin(\frac{1}{n})$, $\cos(\frac{1}{n})$, $\ln(1+\frac{1}{n})$?
- yes → **Let b_n = substitute into a_n**
  $$\cos(\tfrac{1}{n}) \sim 1 - \tfrac{1}{2n^2}$$
  $$\sin(\tfrac{1}{n}) \sim \tfrac{1}{n}$$
  $$\ln(1+\tfrac{1}{n}) \sim \tfrac{1}{n}$$
- no ↓

Try Limit Comparison Test: You must know whether $\sum b_n$ converges or diverges. Calculate $L = \lim\limits_{n \to \infty} \frac{a_n}{b_n}$.
- $0 < L < \infty$ → $\sum a_n$ and $\sum b_n$ do the same thing.
- $L = 0$ → **Does $\sum b_n$ converge?**
  - yes → Converges
  - no → Try a different b_n and repeat or continue on.
- $L = \infty$ → **Does $\sum b_n$ diverge?**
  - yes → diverges
  - no ↓

Does a_n involve $\sin n$ or $\cos n$?
- yes → **Use $-1 \leq \sin n, \cos n \leq 1$ to compare a_n to a b_n without $\sin n$ or $\cos n$. Use Ordinary Comparison Test.**
  - $a_n \leq b_n$ → **Does $\sum b_n$ converge?**
    - yes → converges
    - no → Try a different b_n or continue.
  - $b_n \leq a_n$ → **Does $\sum b_n$ diverge?**
    - yes → diverges
    - no ↓
- no ↓

Does a_n involve something you can integrate?
- yes → **Try Integral Test: Evaluate $\int_1^\infty f(x)\,dx$ where $f(n) = a_n$** → $\int_1^\infty f(x)\,dx$ and $\sum\limits_{n=1}^\infty a_n$ do the same thing.
  - $f(x)$ continuous, positive, non-increasing
- no ↓

You should have discovered the answer by now. You may have to use the Comparison Test in conjunction with the Integral Test when a_n involves transcendental functions. Otherwise you may have to resort to looking at partial sums.