Series/Sequences

Power Series

$\frac{1}{1-\heartsuit} = \sum_{n=0}^{\infty} \heartsuit^n, \forall |\heartsuit| < 1$

$\frac{1}{(1-\heartsuit)^2} = \sum_{n=0}^{\infty} (n+1)\heartsuit^n, \forall |\heartsuit| < 1$

$\ln(1-\heartsuit) = \sum_{n=0}^{\infty} \frac{\heartsuit^{n+1}}{n+1}, \forall |\heartsuit| < 1$

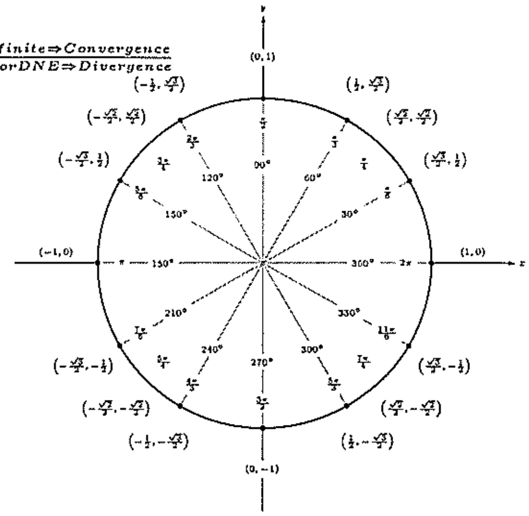
$e^{\heartsuit} = \sum_{n=0}^{\infty} \frac{\heartsuit^n}{n!}, \forall \heartsuit R$

Series

1. ALT. Series
   (a) Use AST or Nth term test
      i. $\lim_{n\to\infty} = 0$ Then it at least conditionally converges
      ii. $\lim_{n\to\infty} \neq 0$ Then it diverges
   (b) If it converges, check Abs. Convergence using ANY prior test
2. If it's always Pos. or Neg., use any Pos. Series Tests.
   (a) Will only be Abs. Convergent or Divergent
3. Not Alternating but also not always Pos. or Neg., test for Abs. Convergence or diverence. Conditional convergence is not an option
4. $\sum \frac{1}{n}$ = harmonic series $\Rightarrow$ diverges
5. $\sum \frac{-1^n}{n} = (-1)^n \frac{1}{n}$ = Alternate harmonic series $\Rightarrow$ Conditionally Converges

Sequences

$\lim_{n\to\infty} a_n = L \begin{cases} L is finite \Rightarrow Convergence \\ \pm\infty or DNE \Rightarrow Divergence \end{cases}$



Derivatives

Trig

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trig

$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$

Hyperbolic Trig

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \text{sech}^2 x$

$\frac{d}{dx}(\text{sech}\, x) = -\text{sech}\, x \tanh x$

$\frac{d}{dx}(\text{csch}\, x) = -\text{csch}\, x \coth x$

$\frac{d}{dx}(\coth x) = -\text{csch}^2 x$

Exponential/Log

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$

$\frac{d}{dx}(\ln |x|) = \frac{1}{x}, x \neq 0$

Identities/Substitution

Trig/Hyperbolic Trig

$\sqrt{a^2 - b^2x^2} \Rightarrow x = \frac{a}{b}\sin\theta$

$\sqrt{a^2 + b^2x^2} \Rightarrow x = \frac{a}{b}\tan\theta$

$\sqrt{b^2x^2 - a^2} \Rightarrow x = \frac{a}{b}\sec\theta$

$\sinh = \frac{1}{2}\left(e^x - e^{-x}\right)$

$\cosh = \frac{1}{2}\left(e^x + e^{-x}\right)$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Exp/log

$e^{-\infty} = 0$

$\pm\ln 0 = \mp\infty$

$\ln e = 1$

$\sum_{n=0}^{\infty} = \frac{x^n}{n!}$

Trig Limits

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 1$

Integrals

Trig

$\int \cos u\,du = \sin u + c$

$\int \sin u\,du = -\cos u + c$

$\int \tan u\,du = \ln|\sec u| + c$

$\int \sec u\,du = \ln|\sec u + \tan u| + c$

$\int \csc u\,du = \ln|\csc u - \cot u| + c$

$\int cot u du = \ln|\sin u| + c$

$\int \sec u \tan u\,du = \sec u + c$

$\int \csc u \cot u\,du = -\csc u + c$

$\int \sec^2 u\,du = \tan u + c$

$\int \csc^2 u\,du = -\cot u + c$

$\int \cot^2 u\,du = -x - \cot u + c$

Inverse Trig

$\int \frac{1}{\sqrt{a^2+u^2}}du = \frac{1}{a}\sin^{-1}\left(\frac{u}{a}\right) + c$

$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$

$\int \frac{1}{u\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + c$

Miscellaneous

$\int \frac{1}{a^2-u^2}du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + c$

$\int \frac{1}{u^2-a^2}du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c$

$\int \sqrt{a^2+u^2}du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln|u + \sqrt{a^2+u^2}| + c$

$\int \sqrt{u^2-a^2}du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln|u + \sqrt{u^2-a^2}| + c$

$\int \sqrt{a^2-u^2}du = \frac{u}{2}\sqrt{a^2-u^2} + \sin^{-1}\left(\frac{u}{a}\right) + c$

Hyperbolic Trig

$\int \sinh u\,du = \cosh u + c$

$\int \cosh u\,du = \sinh u + c$

$\int \tanh u\,du = \ln(\cosh u) + c$

$\int \text{sech}\, u\,du = \tan^{-1}|\sinh u| + c$

$\int \text{sech}\, u \tanh u\,du = -\text{sech}\, u + c$

$\int \text{csch}\, u \coth u\,du = -\text{csch}\, u + c$

$\int \text{sech}^2\, du = \tanh u + c$

$\int \text{csch}^2\, du = -\coth u + c$

Exponential/Log

$\int \frac{1}{x}dx \equiv \int x^{-1} = \ln|x| + c$

$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| + c$

$\int a^u du = \frac{a^u}{\ln a} + c$

$\int e^u du = e^u + c$

$\int ue^u du = (u-1)e^u + c$

$\int \ln u\,du = u\ln(u) - u + c$

$\int \frac{1}{u \ln u}du = \ln|\ln u| + c$

Ind. Forms

$\frac{0}{0}$	$1^0 = 1$
$\frac{\pm\infty}{\pm\infty}$	$0^\infty = 0$
$0 \cdot \infty$	$\infty^\infty = \infty$
$\infty - \infty$	$\infty \cdot \infty = \infty$
0^0	$\infty + \infty = \infty$
∞^0	$\frac{0}{\infty} = 0$
1^∞	$\frac{\infty}{0} = \infty$
	$0^{-\infty} = \pm\infty$

- Geometric Series: $a \neq 0$; $\sum_{i=1}^{\infty} ar^{i-1} = a + ar + ar^2 \cdots \Longrightarrow$ test $\frac{a}{1-r}$
- Nth term test: if $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow \lim_{n\to\infty} a_n = 0$ If using AST and $\lim_{n\to\infty} a_n = 0$ it converges but still need to test for Abs. Convergence or Divergence.
  - $\Leftrightarrow \lim_{n\to\infty} an \neq 0$ or DNE, then it diverges
- Integral Test: if f(x) is; pos., continuous and non-increasing, on $[n, \infty)$ then, $\sum_{k=n}^{\infty} a_n$ converges $\Leftrightarrow \int_n^{\infty} f(x)dx$ converges
- P-Series Test: $\sum_{n=1}^{\infty} \frac{1}{n^p} \Rightarrow p > 1$ converges, $p \leq 1$ diverges
- Limit Comparison Test: Assume $a_n \geq 0$, $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$
  - if $0 < L < \infty$, then $\sum a_n$ and b_n converge or diverge together
  - if $L = 0$ and $\sum b_n$ converges(p-series test) then, $\sum a_n$ converges
  - if $L = \infty$, do the integral test
- Ratio Test and ART: $\sum a_n$ is a series of pos. terms and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = p$
  - $p < 1$ converges | ART $p < 1$ Absolutely converges
  - $p > 1$ or $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$ Diverges | ART $\Rightarrow$ Diverges
  - $p = 1$ inconclusive | ART $\Rightarrow$ More testing needed
  - Steps to help solve:
    1. Plugin n+1 for all n
    2. multiply by the reciprocal of the original function a_n
    3. simplify and solve to get p
  - Hints to factor
    - $(2n)!$ factors to: $(2n)(2n-1)(2n-2)(2n-3)!$ if you wanted to factor this 3 times.

- Ordinary Comparison Test(useful for trig): if $0 < a_n < b_n \forall\, n \geq N$
  1. if $\sum b_n$ converges, so does $\sum a_n$
  2. if $\sum a_n$ diverges, so does $\sum b_n$
  3. **Like the squeeze thm.
- Power Series: Always use the Absolute Ratio Test.
  - What x value makes the fn go to 1?
  - $\lim_{n\to\infty} \frac{xn^2}{n^2} = 1 \cdot |x| \downarrow (-1, 1)$
  - Now plug in -1 and 1 to the original fn to solve for the convergence set
    1. x=0
    2. (-R,R)
    3. Whole Real Line
  - for $\sum_{n=1}^{\infty} a_n x^n$ in $x - a = \sum a_n(x-a)^n$
    1. x=a
    2. interval (a-R, a+R)
    3. whole real line