Name: ____________________

1090-4 Exam #2

April 9, 2002

There are nine questions on the exam. Calculators not allowed.

1. (10 points) True or false:

   (a) $\log_a(a^x) = a^x$.

   (b) $\log_a(xy) = \log_a(x)\log_a(y)$

   (c) Suppose A is a square matrix. Then A^{-1} exists.

   (d) $\log_a(0) = 1$.

   (e) $\log_b(x) = \frac{\log_a(x)}{\log_b(a)}$

2. (15 points) Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 3 \end{pmatrix}.$$

Find A^{-1} (if it exists).

3. (15 points) Executives at Crown Burger are considering a nationwide franchise expansion. They are considering building two different kinds of establishments: one would feature only a take-out menu; the other would be a traditional sit-down restaurant. Market research suggests that they need to limit their initial expansion to 25 new restaurants. Crown's accounting firm suggests that they need to hire between 60 and 220 full-time employees to take advantage of the tax breaks associated with expansion. A take-out restaurant employ 6 full-time employees, and a sit-down restaurant employs 11. Crown estimates that each take-out restaurant will yield a weekly profit of $10,000, while a sit-down one will gross $18,000 weekly. Based on these profit estimates, the Crown executives are trying to decide how many of each kind of restaurant they should launch.

(a) Let x denote the number of new take-out restaurants Crown launches, and let y denote the number of sit-down launches. Based on the information given above, find the system of inequalities that describes the constraints on x and y. (Do not graph or solve the system of inequalities!)

(b) Write down the function you seek to maximize. (You do not need to actually maximize it!)

(c) Describe *in words* how you would proceed to maximize the function in (b). (You do not need to complete (a) and (b) to receive full credit for (c). To obtain maximum partial credit, you should be as detailed as possible in your explanation.)

4. (10 points) Let

$$A = \begin{pmatrix} 3 & 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & -3 \\ 2 & -5 \end{pmatrix}$$

Perform the following matrix computations (when possible).

(a) A^2

(b) B^3

(c) $\det(C)$

5. (10 points) Compute the following:

(a) $\log_2(64)$

(b) $e^{\ln\left(\frac{1}{e^3}\right)}$

(c) $\log(0.001)$

6. (10 points) Graph the following system of inequalities:

$$\begin{aligned} -2x - y &\geq 2 \\ x &> y. \end{aligned}$$

7. (10 points) Suppose

$$\log_a(x) = -2.4 \qquad \log_a(y) = 3 \qquad \log_a(z) = 1.2.$$

Perform the indicated computations:

(a) $\log_a\left(\frac{z}{\sqrt{yz}}\right)$

(b) $\log_a(x^2 z^{100})$

8. (5 points) Simplify $\log_a(\sqrt[3]{x^2+2x+1})$ so that no exponents (greater than one) or radicals appear in your final answer.

9. (15 points) The mouse population in a newly renovated building is observed to obey the following model

$$P(t) = 1000(0.005)^{(0.3^t)},$$

where t is the number of weeks after the renovation is completed.

(a) What is the maximum mouse population in the building?

(b) Solve for t in terms of P.