SOLUTIONS TO 2002 1090 EXAM #1

1.

(a) F	(b) F	(c) F	(d) T	(e) F

2.(a) If $f(x) = 2x^2 + 2x - 3$ and $g(x) = x - 3$, then

$$\begin{aligned}(f \circ g)(x) = f(x-3) &= 2(x-3)^2 + 2(x-3) - 3 \\ &= 2(x^2 - 6x + 9) + 2x - 6 - 3 \\ &= 2x^2 - 12x + 18 + 2x - 9 \\ &= 2x^2 - 10x + 9.\end{aligned}$$

(b) We have

$$(g \circ g)(x) = g(x-3) = (x-3) - 3 = x - 6.$$

(c) We have

$$\begin{aligned}(f \cdot g)(x) &= (2x^2 + 2x - 3)(x - 3) = 2x^3 - 6x^2 + 2x^2 - 6x - 3x + 9 \\ &= 2x^3 - 4x^2 - 9x + 9.\end{aligned}$$

3. (a) The slope-intercept form of the line $5x - 6y = 4$ is $y = \frac{5}{6}x - \frac{2}{3}$, so it has slope $\frac{5}{6}$. Any line perpendicular to it thus has slope $-\frac{6}{5}$. So the line we seek passes through $(3, 1)$ and has slope $-\frac{6}{5}$. Using the point-slope form, we have

$$(y - 1) = -\frac{6}{5}(x - 3)$$

or, after simplifying,

$$y = -\frac{6}{5}x + \frac{23}{5}.$$

(b) The slope-intercept form of $4x - 2y = 6$ is given by $y = 2x - 3$, and thus has slope 2. Any line parallel to it also has slope 2. Thus we seek a line through $(6, -4)$ with slope 2. Using the point-slope form, we have

$$(y - (-4)) = 2(x - 6)$$

or, after simplifying,

$$y = 2x - 16.$$

4.(a) If we add -2 times the first equation to the second we get

$$-y = 5,$$

So $y = -5$. Plugging back in, we quickly find $x = 1$.

(b) Adding -3 times the second to the third gives

$$2z = -2,$$

so $z = -1$. Plugging back into the second equation gives $y = 12$. Plugging both these values back into the first equation leads quickly to $x = 4$.

5. Set $R(x) = C(x)$ and solve for x. We have

$$16 + 15x + \frac{2}{5}x^2 = 25x - \frac{3}{5}x^2$$

Multiplying through by 5 gives

$$80 + 75x + 2x^2 = 125x - 3x^2$$

or

$$5x^2 - 50x + 80 = 0.$$

Dividing by 5 gives

$$x^2 - 10x + 16 = 0.$$

This factors as

$$(x - 8)(x - 2) = 0.$$

So the break-even points are at 2 and 8 units of production.

6(a). This one factors as $(x - 5)(x + 1) = 0$. So the two solutions are $x = 5$ and $x = -1$.

(b) This is a parabola which opens upward, which crosses the x-axis at -1 and 5, and which has a vertex at $(2, -9)$.

7.(a) We are given that $P(30) = P(100) = 0$. So

$$0 = P(30) = -(30)^2 + 30b + c$$

and

$$0 = P(100) = -(100)^2 + 100b + c.$$

So to find a and b we have to solve the system of two equations:

$$-900 + 30b + c = 0$$
$$-10000 + 100b + c = 0.$$

Subtracting them gives

$$9100 - 70b = 0.$$

So $b = \frac{910}{7} = 130$. Plugging back in we get $c = -3000$.

(b) The profit function is a parabola that opens downward. So it is maximized at its vertex, the x-coordinate of which is at $\frac{-b}{2 \cdot -1} = b/2 = 65$. So Eddie should charge \$65 per ticket.