MATH 1010-2: PRACTICE EXAM #2

1 (10 points). Solve the following equation for x:

$$\sqrt[3]{2x+1} + 2 = 5.$$

Solution. First isolate the radical by subtracting 2 from both sides:

$$\sqrt[3]{2x+1} = 3.$$

Now cube both sides to get

$$2x + 1 = 27.$$

So

$$2x = 26,$$

and $\boxed{x = 13}$.

2 (10 points). Solve the following equation for x:

$$3x^2 + 4x - 2 = 2x^2 + 7x + 8.$$

Solution. Bring everything onto the left-hand side to get

$$x^2 - 3x - 10 = 0.$$

This is one that factors as

$$(x-5)(x+2) = 0.$$

So either $x - 5 = 0$ of $x + 2 = 0$. Thus $\boxed{x = 5}$ or $\boxed{x = -2}$. Alternatively you could have arrived at this answer by either completing the square or using the quadratic formula.

3 (15 points). Graph the following system of linear inequalities:

$$\begin{aligned} x + y &\le 3 \\ x - 1 &\le 1. \end{aligned}$$

Clearly label any vertices in your graph.

Solution. The graph consists of two solid lines meeting at $(2, 1)$. One line is a vertical line though $(2, 0)$. The other has slope -1 and y-intercept 3. The region which is to the left of *both* lines is shaded.

4 (10 points).

(a) **Solution.** We have

$$\det \begin{pmatrix} 1 & -2 \\ 3 & 5 \end{pmatrix} = (1)(5) - 3(-2) = 5 + 6 = \boxed{11}.$$

(b) Write $(3 + 2i)(2 - 7i)$ as a complex number in standard form.

Solution. We use the distributive law (FOIL) and the fact that $i^2 = -1$ to get

$$\begin{aligned} (3 + 2i)(2 - 7i) &= 6 - 21i + 4i - 14i^2 \\ &= 6 + (4 - 21)i - 14(-1) \\ &= 6 - 17i + 14 \\ &= \boxed{20 - 17i}. \end{aligned}$$

5 (10 points). Simplify

$$(3 - 7x + 8x^2) - [(x+3)^2 - 2(x-3)]$$

Write your answer as a polynomial in standard form.

Solution. We use the order of operations on the second term to obtain

$$(3 - 7x + 8x^2) - [x^2 + 6x + 9 - 2x + 6].$$

Grouping like terms we get

$$(3 - 7x + 8x^2) - [x^2 + 4x + 15]$$

or

$$3 - 7x + 8x^2 - x^2 - 4x - 15.$$

Again grouping like terms we get

$$\boxed{7x^2 - 11x - 12}.$$

6 (10 points). Perform the indicated operation and simplify:

$$\frac{-x}{x+3} + \frac{2x+1}{x+7}.$$

Solution. The common denominator in this case is $(x+3)(x+7)$. So we rewrite the original expression as

$$\frac{-x}{x+3} \cdot \frac{x+7}{x+7} + \frac{2x+1}{x+7} \cdot \frac{x+3}{x+3}.$$

Simplifying we obtain

$$\frac{-x^2 - 7x + 2x^2 + x + 6x + 3}{(x+3)(x+7)}$$

or

$$\boxed{\frac{x^2+3}{(x+3)(x+7)}}.$$

7 (10 points). Rationalize the denominator of the following expression and simplify:

$$\frac{\sqrt{5}+1}{1-\sqrt{2}}.$$

Solution. We mulitply by the conjugate $1+\sqrt{2}$ and FOIL as follows:

$$\begin{aligned}
\frac{\sqrt{5}+1}{1-\sqrt{2}} &= \frac{\sqrt{5}+1}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} \\
&= \frac{(\sqrt{5}+1)(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} \\
&= \frac{\sqrt{5}+\sqrt{10}+1+\sqrt{2}}{1-(\sqrt{2})^2} \\
&= \frac{\sqrt{5}+\sqrt{10}+1+\sqrt{2}}{1-2} \\
&= \frac{\sqrt{5}+\sqrt{10}+1+\sqrt{2}}{-1} \\
&= -(\sqrt{5}+\sqrt{10}+1+\sqrt{2}) \\
&= \boxed{-\sqrt{5}-\sqrt{10}-1-\sqrt{2}}.
\end{aligned}$$

8 (10 points). Simplify the following complex fraction:

$$\frac{\left(\frac{x+3}{x+5}\right)}{\left(\frac{1}{x^2-25}\right)}.$$

Solution. Invert and multiply to obtain

$$\frac{(x+3)(x^2-25)}{x+5}.$$

Since $x^2-25=(x+5)(x-5)$ we have

$$\frac{(x+3)(x+5)(x-5)}{x+5}.$$

Finally, we can cancel to arrive at

$$\boxed{(x+3)(x-5) \qquad x \neq -5}.$$

9 (5 points). Using fractional exponents, rewrite the following expression without radicals and simplify:

$$\frac{(3u-2v)^{2/3}}{\sqrt{(3u-2v)^3}}.$$

$$\frac{(3u-2v)^{2/3}}{\sqrt{(3u-2v)^3}}.$$

Solution. We first remove the square-root in the denominator

$$\frac{(3u-2v)^{2/3}}{[(3u-2v)^3]^{1/2}}.$$

Since successive exponents multiply, we get

$$\frac{(3u-2v)^{2/3}}{(3u-2v)^{3/2}}.$$

Now we subtract exponents to obtain

$$(3u-2v)^{2/3-3/2} = \boxed{(3u-2v)^{-5/6}}.$$

10 (10 points). Rewrite the following expression without any fractional exponents, negative exponents, *or* radicals:

$$\frac{8^{1/2}\sqrt{x^3}}{8^{1/6}\sqrt{x^9}}.$$

Solution. First remove the square-roots

$$\frac{8^{1/2}(x^3)^{1/2}}{8^{1/6}(x^9)^{1/2}}$$

and multiply the successive exponents to get

$$\frac{8^{1/2}x^{3/2}}{8^{1/6}x^{9/2}}.$$

Now subtract the exponents

$$8^{1/2-1/6}x^{3/2-9/2}$$

to obtain

$$8^{1/3}x^{-3}$$

and finally

$$\boxed{\frac{2}{x^3}}.$$

11 (10 points). Solve for x by any method you choose:

$$(x+2)^2 - 15 = 0.$$

Solution. Adding 15 to both sides gives

$$(x+2)^2 = 15.$$

Taking square-roots, we have

$$x + 2 = \pm\sqrt{15},$$

and so

$$\boxed{x = -2 \pm \sqrt{15}}.$$