

## MATH 1010-2: PRACTICE FINAL<sup>1</sup>

December, 2010

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1. Solve for  $x$ :

$$7x - 2(x - 3) = 26.$$

**Solution.** Distribute the  $-2$  through (being careful with signs!) to obtain

$$7x - 2x + 6 = 26,$$

or

$$5x = 20.$$

So  $x = 4$ .

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1\*. Solve for  $x$ :

$$\frac{x}{x-1} = 6.$$

**Solution.** Clear the denominator by multiplying both sides by  $x - 1$  to obtain

$$x = 6(x - 1) = 6x - 6.$$

So

$$5x = 6,$$

and  $x = 6/5$ .

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1\*\*. Solve for  $x$ :

$$-2(2 - x) - 3x = 16.$$

**Solution.** Distribute the  $-2$  through (again being careful with the signs!) to obtain

$$-4 + 2x - 3x = 16.$$

So

$$-x = 20,$$

and  $x = -20$ .

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2. Simplify

$$\frac{\frac{1}{5} - \frac{1}{6}}{\frac{2}{3}}$$

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<sup>1</sup>There are more problems on this practice final than on the actual final exam. To get a feel for the length of the actual exam, you should choose one problem numbered 1, one problem numbered 2, and so on.

**Solution.** We work inside the parenthesis first, and find a common denominator. The best we can do is  $30 = 5 \cdot 6$ . So we have

$$\begin{aligned}\frac{\frac{1}{5} - \frac{1}{6}}{\frac{2}{3}} &= \frac{\left(\frac{1}{5} \cdot \frac{6}{6} - \frac{1}{6} \cdot \frac{5}{5}\right)}{\frac{2}{3}} \\ &= \frac{\frac{6}{30} - \frac{5}{30}}{\frac{2}{3}} \\ &= \frac{\frac{1}{30}}{\frac{2}{3}}.\end{aligned}$$

Now we invert and multiply to get

$$\frac{1}{30} \cdot \frac{3}{2} = \frac{3}{60} = \frac{1}{20},$$

which is our final answer.

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2\*. Simplify

$$\frac{(2 - \frac{3^4}{9})}{\frac{1}{2}}$$

**Solution.** Using the order of operation rules (and inverting and multiplying at the last step), we have

$$\begin{aligned}\frac{(2 - \frac{3^4}{9})}{\frac{1}{2}} &= \frac{2 - \frac{81}{9}}{\frac{1}{2}} \\ &= \frac{2 - 9}{\frac{1}{2}} \\ &= -7 \cdot \frac{2}{1} = -14.\end{aligned}$$

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2\*\*. Simplify

$$\frac{1}{1 - 7/5}$$

**Solution.** We first find a common denominator downstairs and then invert and multiply:

$$\begin{aligned}\frac{1}{1 - 7/5} &= \frac{1}{5/5 - 7/5} \\ &= \frac{1}{-2/5} \\ &= 1 \cdot (-5/2) = -5/2.\end{aligned}$$

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3. A hiker travels 3 miles east before turning and traveling due south to his destination. If he ends up 5 miles (as the crow flies) from his starting point, how far did he travel on the southbound leg of his hike?

**Solution.** The problem describes a right triangle with one leg equal to 3 and with hypotenuse equal to 5. We seek the length of the other leg. By the Pythagorean Theorem, it equals

$$\sqrt{5^2 - 3^2} = 4.$$

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3\*. The KSL tower is 80 meters tall. A 100 meter guy wire is stretched taut from the top of the tower to an anchor on the ground. How far is the anchor from the base of the tower?

**Solution.** By the Pythagorean Theorem, the length we seek is

$$\sqrt{100^2 - 80^2} = \sqrt{3600} = 60.$$

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3\*\*. Find the distance between  $(-1, -2)$  and  $(3, -5)$ .

**Solution.** We use the distance formulas with  $(x_1, y_1) = (-1, -2)$  and  $(x_2, y_2) = (3, -5)$ ,

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(-1 - 3)^2 + (-2 - (-5))^2} \\ &= \sqrt{(-4)^2 + (3)^2} \\ &= \sqrt{16 + 9} = 5. \end{aligned}$$

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4. Find the equation of the line which passes through  $(1, -2)$  and which is parallel to the line  $2x + y = 3$ . Write your answer in slope-intercept form, and sketch its graph.

**Solution.** The line  $2x + y = 3$  in slope-intercept form is  $y = -2x + 3$ , and thus it has slope  $-2$ . Thus the line we are seeking passes through  $(1, -2)$  and has slope  $-2$ , and so it is

$$y - (-2) = -2(x - 1).$$

Simplifying and converting to slope-intercept form gives

$$y = -2x.$$

This is a line through the origin with slope  $-2$ . (I'll leave the graph to you.)

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4\*. Find the equation of the line which passes through  $(0, 2)$  and which is perpendicular to the line  $2y = 1 - x$ . Write your answer in slope-intercept form, and sketch its graph.

**Solution.** The line  $2y = 1 - x$  in slope-intercept form is  $y = -\frac{1}{2}x + \frac{1}{2}$ , and thus has slope  $-\frac{1}{2}$ . Any line perpendicular to it has slope equal to the negative reciprocal of  $-\frac{1}{2}$ , namely 2. So the line we seek has slope 2 and passes through  $(0, 2)$ . So it is given by

$$y - 2 = 2(x - 0),$$

or, in slope-intercept form,

$$y = 2x + 2.$$

Once again I'll leave the graph to you.

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4\*\*. Find the equation of the line which passes through  $(2, -3)$  and which has the same slope as the line passing through  $(2, 3)$  and  $(4, -5)$ . Write your answer in slope-intercept form, and sketch its graph.

**Solution.** The line through  $(2, 3)$  and  $(4, -5)$  has slope

$$\frac{-5 - 3}{4 - 2} = -4.$$

So the line we seek has slope  $-4$  and passes through  $(2, -3)$ . Hence it is given by

$$y - (-3) = -4(x - 2).$$

In slope-intercept form this is

$$y = -4x + 5.$$

I'll leave the graph to you.

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5. Perform the indicated operations and simplify:

$$(x + 1)^2 - (x + 2)^2 + 3.$$

**Solution.** Remember  $(x + 1)^2 \neq x^2 + 1!!$  Instead we rewrite the original expression as

$$(x + 1)(x + 1) - (x + 2)(x + 2) + 3.$$

FOILING out gives

$$x^2 + 2x + 1 - (x^2 + 4x + 4) + 3,$$

and distributing the minus sign gives

$$x^2 + 2x + 1 - x^2 - 4x - 4 + 3.$$

Finally combining like terms gives

$$-2x.$$

The degree is 1 and the leading coefficient is  $-2$ .

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5\*. Perform the indicated operations and simplify:

$$(x - 1)(x^2 - 2x + 3) - (x + 2).$$

**Solution.** We distribute to get

$$x(x^2 - 2x + 3) - (x^2 - 2x + 3) - x - 2,$$

or

$$x^3 - 2x^2 + 3x - x^2 + 2x - 3 - x - 2,$$

and combining like terms gives

$$x^3 - 3x^2 + 4x - 5.$$

The degree is 3 and the leading coefficient is 1.

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6. Perform the indicated operations and simplify:

$$\frac{x^2 - 4x - 5}{x^2 - 1} \cdot \frac{x^2 + 3x + 2}{x^2 + 10x + 25}.$$

**Solution.** We have

$$\frac{x^2 - 4x - 5}{x^2 - 1} \cdot \frac{x^2 + 3x + 2}{x^2 + 10x + 25} = \frac{(x^2 - 4x - 5)(x^2 + 3x + 2)}{(x^2 - 1)(x^2 + 10x + 25)}.$$

To simplify we must factor as much as possible

$$\frac{(x - 5)(x + 1)(x + 2)(x + 1)}{(x + 1)(x - 1)(x + 5)(x + 5)}$$

and then cancel one of the  $x + 1$  factors to get

$$\frac{(x - 5)(x + 2)(x + 1)}{(x + 5)(x + 5)(x - 1)};$$

note that we have masked a “bad point” and must stipulate  $x \neq -1$ .

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6\*, Perform the indicated operations and simplify:

$$\frac{x}{x - 3} + \frac{1}{x - 4} - \frac{x}{x + 4}$$

**Solution.** We have to find a common denominator. The best we can do is  $(x - 3)(x - 4)(x + 4)$ . So the original expression becomes

$$\frac{x}{x - 3} \cdot \frac{(x - 4)(x + 4)}{(x - 4)(x + 4)} + \frac{1}{x - 4} \cdot \frac{(x - 3)(x + 4)}{(x - 3)(x + 4)} - \frac{x}{x + 4} \cdot \frac{(x - 3)(x - 4)}{(x - 3)(x - 4)}$$

or

$$\frac{x(x + 4)(x - 4)}{(x - 3)(x - 4)(x + 4)} + \frac{(x - 3)(x + 4)}{(x - 3)(x - 4)(x + 4)} - \frac{x(x - 3)(x - 4)}{(x - 3)(x - 4)(x + 4)}.$$

We have a common denominator, so we can add the numerators to obtain

$$\frac{x(x + 4)(x - 4) + (x - 3)(x + 4) - x(x - 3)(x - 4)}{(x - 3)(x - 4)(x + 4)}.$$

We can simplify the numerator by FOILING and further distributing to get

$$\frac{x(x^2 - 16) + x^2 + x - 12 - x(x^2 - 7x + 12)}{(x - 3)(x - 4)(x + 4)}$$

or

$$\frac{x^3 - 16x + x^2 + x - 12 - x^3 + 7x^2 - 12x}{(x - 3)(x - 4)(x + 4)}.$$

Combining like-terms in the numerator, we get

$$\frac{8x^2 - 27x - 12}{(x - 3)(x - 4)(x + 4)}.$$

We could multiply the denominator, but we don't need to.

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7. Rewrite using rational exponents and simplify so that no fractional exponents or radicals appear in your final answer:

$$\frac{\sqrt[4]{16x^6}}{\sqrt{x^3}}.$$

**Solution.** We convert to obtain

$$\frac{(16x^6)^{1/4}}{(x^3)^{1/2}},$$

and use the rule of exponents to arrive at

$$\frac{16^{1/4}x^{6/4}}{x^{3/2}} = 2x^{6/4-3/2} = 2.$$

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8. Simplify so that no fractional exponents (or radicals) appear in your final answer:

$$\frac{x^{2/3}}{x^{-1/3}} \cdot (x^6)^{1/3}.$$

**Solution.** Use the rules of exponents to obtain

$$x^{2/3-(-1/3)}x^{6/3} = x^1x^2 = x^3.$$

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9. Solve by any method you choose:

$$x^2 + 11x + 28 = 0.$$

**Solution.** This one factors nicely as

$$(x + 7)(x + 4) = 0,$$

so either  $x + 7 = 0$  or  $x + 4 = 0$ . Thus  $x = -4$  or  $x = -7$ . (You could have used the quadratic formula or could have completed the square to obtain the same answer.)

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9\*. Solve by any method you choose:

$$x^2 + x - 7 = 0.$$

**Solution.** This one doesn't factor nicely. So (for instance) we can apply the quadratic formula with  $a = 1$ ,  $b = 1$ , and  $c = -7$  to get

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-7)}}{2(1)} = \frac{-1 \pm \sqrt{29}}{2}.$$

(You could also have completed the square to get the final answer.)

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9\*\*. Solve by any method you choose:

$$x^2 + 2x = 9.$$

**Solution.** One possibility is to complete the square by adding 1 to both sides to obtain

$$x^2 + 2x + 1 = 9 + 1,$$

or

$$(x + 1)^2 = 10.$$

So

$$x + 1 = \sqrt{10} \quad \text{or} \quad x + 1 = -\sqrt{10}.$$

So

$$x = -1 \pm \sqrt{10},$$

which one could have also obtained by using the quadratic formula.

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9\*\*\*. Solve by any method you choose:

$$-x^2 + 2x = 2(x^2 - x) - 5.$$

**Solution.** Bring everything on one side to obtain

$$3x^2 - 4x - 5 = 0.$$

This one doesn't factor nicely, so we apply the quadratic formula with  $a = 3$ ,  $b = -4$ , and  $c = -5$  to get

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)} = \frac{4 \pm \sqrt{76}}{6}$$

which we could simplify as

$$x = \frac{4 \pm 2\sqrt{19}}{6} = \frac{2 \pm \sqrt{19}}{3}.$$

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10. Solve for  $x$ :

$$2 - \sqrt{x - 3} = 0.$$

**Solution.** Isolate the radical to obtain

$$\sqrt{x - 3} = 2.$$

Squaring both sides we get

$$x - 3 = 4,$$

and so  $x = 7$ . This solution checks out.

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10\*. Solve for  $x$ :

$$\sqrt{8 - 2x} = x.$$

**Solution.** Squaring both sides we obtain

$$8 - 2x = x^2.$$

Now we bring everything on one side to get

$$x^2 + 2x - 8 = 0.$$

This one factors as

$$(x + 4)(x - 2) = 0,$$

and so  $x = -4$  or  $x = 2$ . Plugging back in (as we must with radical equations!), we see that  $x = -4$  is not a valid solution. So  $x = 2$  is the only solution.

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10\*\*. Solve for  $x$ :

$$\sqrt{x - 1} = 1 + \sqrt{x - 7}.$$

**Solution.** We cannot isolate both radical simultaneously, so we square the given equation (and FOIL the right side) to get

$$\begin{aligned}(\sqrt{x - 1})^2 &= (1 + \sqrt{x - 7})^2 \\ x - 1 &= 1 + \sqrt{x - 7} + \sqrt{x - 7} + (x - 7) \\ x - 1 &= 1 + 2\sqrt{x - 7} + (x - 7).\end{aligned}$$

Now we can simplify and isolate the radical to obtain

$$5 = 2\sqrt{x - 7}.$$

Squaring both sides yields,

$$25 = 4(x - 7)$$

or

$$25 = 4x - 28,$$

which we quickly solve to get

$$x = 53/4.$$

This solution checks out.

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11. Eighty percent of a number equals half the number plus 10. What is the number?

**Solution.** Let  $x$  be the number we seek. The first sentence of the problem translates into

$$0.8x = 0.5x + 10.$$



So

$$0, 3x = 10,$$

or

$$x = \frac{10}{0.3} = \frac{100}{3}.$$

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11\*. A small rectangular garden plot is 800 square feet in area. If the length of the plot is twice its width, what is the width of the plot?

**Solution.** Let  $w$  be the width. Then the length is  $2w$  and so

$$w(2w) = 800.$$

Thus

$$2w^2 = 800,$$

or

$$w^2 = 400,$$

and  $w = 20$ .

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12. Solve the following system of equations by any method you wish:

$$x - 2y = 13$$

$$2x - y = 11.$$

**Solution.** There are a number of ways to do this. The answer is  $x = 3$  and  $y = -5$ .

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12\*. Solve the following system of equations by any method you wish:

$$x - 2y + 3z = 22$$

$$5y + 3z = 26.$$

$$3y - 2z = -11.$$

**Solution.** This one is more complicated. But note that the last two equations are two equations in two unknowns. So, setting aside the first equation for a moment, we can solve the second and third (by any number of means) to get  $y = 1$  and  $z = 7$ . Now plug those into the first to get

$$x - 2(1) + 3(7) = 22,$$

which we can quickly solve to get  $x = 3$ .

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13. Simplify as much as possible:

$$\sqrt[5]{64x^5y^6}.$$

**Solution.** By grouping in groups of five as much as possible, we obtain

$$2xy\sqrt[5]{2y}$$

as our answer.

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13\*. Simplify as much as possible:

$$\sqrt[3]{\frac{16z^3}{y^6}}.$$

**Solution.** We break the radical apart over the fraction to get

$$\frac{\sqrt[3]{16z^3}}{\sqrt[3]{y^6}}.$$

By grouping in groups of three as much as possible, we obtain

$$\frac{2z\sqrt[3]{2}}{y^2}.$$

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13\*\*. Simplify as much as possible:

$$\sqrt{4e^{6x}y^3}.$$

**Solution.** By grouping in groups of two as much as possible, we obtain

$$\begin{aligned}\sqrt{4e^{6x}y^3} &= \sqrt{2^2(e^{3x})^2y^2y} \\ &= 2e^{3x}y\sqrt{y}.\end{aligned}$$

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14. Solve for  $x$ :

$$\frac{500}{3x+5} = \frac{50}{x-3}.$$

**Solution.** Multiplying both sides by  $(x-3)(3x+5)$  to clear denominators (or “cross multiplying”) gives

$$500(x-3) = 50(3x+5)$$

or

$$350x = 1750.$$

So

$$x = \frac{1750}{350} = \frac{175}{35} = 5.$$

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14\*. Solve for  $x$ :

$$x - \frac{24}{x} = 5.$$

**Solution.** Clear denominators by multiplying through by  $x$  to obtain

$$x^2 - 24 = 5x.$$

Bringing everything on one side of the equation gives

$$x^2 - 5x - 24 = 0.$$

This one factors as

$$(x - 8)(x + 3) = 0,$$

and so  $x = 8$  or  $x = -3$ .

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14\*\*. Find all solutions of:

$$\frac{1}{y+2} - \frac{1}{y+4} = \frac{2}{15}$$

**Solution.** We need to clear the denominators by multiplying both sides by  $15(y+2)(y+4)$ . We get

$$15(y+2)(y+4) \left( \frac{1}{y+2} - \frac{1}{y+4} \right) = 15(y+2)(y+4) \left( \frac{2}{15} \right).$$

Distributing, we get

$$15(y+4) - 15(y+2) = 2(y+2)(y+4).$$

Distributing further, we get

$$15y + 60 - 15y - 30 = 2(y^2 + 6y + 8)$$

or

$$30 = 2y^2 + 12y + 16.$$

Bringing everything to the right side gives

$$0 = 2y^2 + 12y - 14.$$

We can divide both sides by 2 to get

$$0 = y^2 + 6y - 7.$$

This factors as

$$0 = (y+7)(y-1).$$

So we conclude  $y = -7$  or  $y = 1$ . Both solutions check out.

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15. Find the domain of

$$f(x) = \frac{1}{(x-2)(x+3)}$$

**Solution.** Remember that the domain consists of all numbers which can be input to  $f$ . We get into trouble if we divide by zero. The only way that can happen is if  $x = 2$  or  $x = -3$ . Thus the domain consists of all real numbers except for  $x = 2$  and  $x = -3$ .

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15\*. Find the domain of

$$f(x) = \frac{1}{x^2 - 8x + 15}$$

**Solution.** Again we get into trouble only when the denominator is zero,

$$x^2 - 8x + 15 = 0.$$

This factors as  $(x - 5)(x - 3) = 0$ . So we get into trouble when  $x = 3$  or  $5$ . Thus the domain of  $f$  consists of all real numbers except  $x = 3$  and  $x = 5$ .

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15\*\*. Find the domain of

$$f(x) = \log_{10}(5x - 7).$$

**Solution.** Remember that  $\log_a$  of a negative number (or zero) does not make sense. So we get into trouble when

$$5x - 7 \leq 0,$$

that is when

$$x \leq \frac{7}{5}.$$

These are the places we get into trouble. Thus the domain consists of all  $x$  for which  $x > \frac{7}{5}$ .

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16. You and your neighbor can build a 20 foot section of fence in 6 hours. Working on your own, it would take you 14 hours. How long would it take your neighbor if he were to work on his own?

**Solution.** Let  $x$  be the number of hours it would take your neighbor to build the fence. The information in the problem tells you:

Your work rate: 1 fence/14 hours

Your neighbor's work rate: 1 fence/ $x$  hours.

The final piece of information is that you can build one fence in 6 hours working together:

$$1 \text{ fence} = 6 \text{ hours} (1 \text{ fence}/14 \text{ hours} + 1 \text{ fence}/x \text{ hours})$$

or

$$1 = 6 \left( \frac{1}{14} + \frac{1}{x} \right).$$

Distributing the 6 through gives

$$1 = \frac{6}{14} + \frac{6}{x}.$$

Multiplying both sides by  $14x$  to clear denominators, we get

$$14x = 14x \left( \frac{6}{14} + \frac{6}{x} \right)$$

or

$$14x = 6x + 84.$$

We quickly solve for  $x$  to get

$$x = 84/8 = 10.5.$$

So it would take your neighbor 10.5 hours to complete the fence on his own.

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16\*. There are two kinds of tickets sold to the Ballet West production of the Nutcracker. Adult tickets cost \$21 and kids tickets cost \$8. If there are twice as many children in the sold out 2400-seat theatre, how much money did the box office collect?

**Solution.** Let  $x$  be the number of adult tickets and  $y$  be the number of kids tickets. We seek the total sales figure, namely

$$\text{total sales} = 21x + 8y.$$

We also know there are the number of kids tickets is twice the number of adult tickets,

$$y = 2x.$$

Finally we know the number of tickets sold (kids and adults) is 2400,

$$x + y = 2400.$$

Substituting  $y = 2x$ , we get

$$x + 2x = 2400,$$

and so  $3x = 2400$  and  $x = 800$ . Thus  $y = 2(800) = 1600$ . So the total sales are

$$\text{total sales} = 21(800) + 8(1600) = 16800 + 12800 = 29600.$$

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17. If  $f(x) = x^2 + 2x + 1$  and  $g(x) = x - 1$ , compute  $(f \circ g)(x)$ .

**Solution.** We have

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x - 1) \\ &= (x - 1)^2 + 2(x - 1) + 1 \\ &= x^2 - 2x + 1 + 2x - 2 + 1 \\ &= x^2.\end{aligned}$$

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17\*. If  $f(x) = \log_3(3x - 1)$  and  $g(x) = 3^x + \frac{1}{3}$ , compute  $(f \circ g)(x)$ .

**Solution.** We have

$$\begin{aligned}(f \circ g)(x) &= f\left(3^x + \frac{1}{3}\right) \\ &= \log_3\left(3\left(3^x + \frac{1}{3}\right) - 1\right) \\ &= \log_3(3^{x+1} + 1 - 1) \\ &= \log_3(3^{x+1}) \\ &= x + 1.\end{aligned}$$

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17\*\*. If  $f(x) = x^2 + 2$  compute  $(f \circ f)(x)$ .

**Solution.** We have

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= f(x^2 + 2) \\ &= (x^2 + 2)^2 + 2 \\ &= x^4 + 4x^2 + 4 + 2 \\ &= x^4 + 4x^2 + 6.\end{aligned}$$

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18. Given that  $\log_a(x) = 0.5$  and  $\log_a(y) = 0.25$ , compute  $\log_a(x^2y^3)$ .

**Solution.** Using the rules of logs, we have

$$\begin{aligned}\log_a(x^2y^3) &= \log_a(x^2) + \log_a(y^3) \\ &= 2\log_a(x) + 3\log_a(y) \\ &= 2(0.5) + 3(0.25) \\ &= 1 + 0.75 = 1.75.\end{aligned}$$

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18\*. Using the property of logs, simplify  $\log_5(50) - \log_5(10)$ .

**Solution.** We have

$$\log_5(50) - \log_5(10) = \log_5\left(\frac{50}{10}\right) = \log_5(5) = 1.$$

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18\*\*. Using properties of logs, condense the following expression:  $\log_3(2) + \frac{1}{2}\log_3(y)$ .

**Solution.** We have

$$\begin{aligned}\log_3(2) + \frac{1}{2}\log_3(y) &= \log_3(2) + \log_3(y^{1/2}) \\ &= \log_3(2y^{1/2}) \\ &= \log_3(2\sqrt{y}).\end{aligned}$$

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19. Sketch the graph of  $f(x) = 2^x + 3$ . Clearly label any  $x$ - and  $y$ -intercepts.

**Solution.** The graph in question is the graph of  $2^x$  (which we gave in class) shifted up 3 units. Thus our graph blows up in the positive direction, and (in the negative direction) has a horizontal asymptote at  $y = 3$ . It doesn't cross the  $x$ -axis, so there are no  $x$ -intercepts. To find the  $y$  intercept, we plug in  $x = 0$  to get  $2^0 + 3 = 1 + 3 = 4$ . I'll leave the sketching to you.

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19\*. Sketch the graph of  $g(x) = x^2 - 3x - 4$ . Clearly label any  $x$ - and  $y$ -intercepts.

**Solution.** This is a parabola which opens upward. The  $x$  coordinate of its vertex is

$$x = -b/2a = -(-3)/2(1) = 3/2.$$

We plug in to find the  $y$  coordinate,

$$y = (3/2)^2 - 3(3/2) - 4 = 9/4 - 9/2 + 4 = 9/4 - 18/4 + 16/4 = -25/4.$$

So the vertex is at  $(3/2, -25/4)$ . The  $y$ -intercept is obtained by plugging in  $x = 0$ , so it is at  $-4$ . Finally, to find the  $x$ -intercepts, we have to set  $y = 0$  and solve

$$x^2 - 3x - 4 = 0.$$

This one factors as  $(x - 4)(x + 1) = 0$ . So the  $x$ -intercepts are at  $x = 4$  or  $x = -1$ . I'll leave the sketching to you.

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20. A \$1000 investment is made in a trust fund with an annual interest rate of 10% compounded continuously. After  $t$  years, the value of the fund is

$$P(t) = 1000e^{t/10}$$

How long will it take the investment to double in value? You may leave logarithms and exponentials in your answer.

**Solution.** We want to find the time it takes for the investment to double to 2000. So we want to solve for  $t$  in

$$2000 = 1000e^{t/10}.$$

Dividing by 1000, we get

$$2 = e^{t/10}.$$

By definition, we have

$$t/10 = \ln(2)$$

or

$$t = 10 \ln(2),$$

about 6.9 years.

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20\*. A radioactive element has a half life of 1000 years. Fifty pounds of the element were buried in the West Desert in 2010. How much of the substance will remain in the year 5010?

**Solution.** In 3010 (after 1000 years), the mass of the substance will have halved to 25 pounds. In 4010 (after another 1000 years) it will have halved again to  $25/2 = 12.5$  pounds. Finally, in 5010 it

will have halved again to  $12.5/2 = 6.25$  pounds. This is our final answer. Alternatively, you could have used the half-life formula to conclude the mass would be

$$50 \left(\frac{1}{2}\right)^{3000/1000} = 50 \left(\frac{1}{2}\right)^3 = 50 \left(\frac{1}{8}\right) = \frac{50}{8} = 6.25.$$