University of Utah
Math 1220, Fall 2007
Name: Solutions.

Quiz # 5
Time: 15 minutes

Please try to carefully explain/justify the steps leading to your solutions.

Part 1: (4 points) Give an example of a sequence (a_n) which tends to 0 but such that the series $(\sum_n a_n)$ diverges. The basic example is $a_n = \frac{1}{n}$, which gives the harmonic series $\left(\sum_{n=1}^{\infty} \frac{1}{n}\right)$ (which diverges as a p-series with $p=1$).

Part 2: (16 points, 4 for each series) For each of the following series, determine if it converges or diverges, and justify your answer (just saying "converges" or "diverges" will not be accepted).

$$(a)\ \left(\sum_{n=1}^{\infty} \frac{n!}{2^n}\right)\quad (b)\ \left(\sum_{n=1}^{\infty} \frac{4n+3}{3n^3+4n+16}\right)\quad (c)\ \left(\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\right)\quad (d)\ \left(\sum_{n=1}^{\infty} \frac{5^n}{99^n}\right)$$

(a) Ratio Test: let $a_n = \frac{n!}{2^n}$. Then $\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!} = \frac{n+1}{2} \xrightarrow[n\to\infty]{} +\infty$
Therefore the series diverges.

(b) Limit comparison Test: The leading term in the numerator is $4n$, the leading term in the denominator is $3n^3$. Therefore the given series converges $\Leftrightarrow$ $\left(\sum \frac{4n}{3n^3}\right)$ converges. Now the latter converges because it is a p-series with $p=2$ $\left(\frac{n}{n^3} = \frac{1}{n^2}\right)$.

(c) converges by the Alternating Series Test, because $\frac{1}{\sqrt{n}}$ decreases to 0.
(it wasn't asked, but this series is not absolutely convergent: $\left(\sum \frac{1}{\sqrt{n}}\right)$ diverges, p-series with $p=\frac{1}{2}$.)

(d) is a geometric series of ratio $\frac{5}{99} < 1$, therefore it converges.