University of Utah
Math 2210, Fall 2008
Name: Key

Quiz # 5
Time: 15 minutes

(1): (15 points) Evaluate the integral:

$$I = \int_0^1 \int_0^{2-2x} \int_0^{4-4x-2y} dz\,dy\,dx.$$

(2): (5 points) Describe and sketch a solid whose volume is given by this integral.

(1) Start with the inner integral: $\int_0^{4-4x-2y} dz = 4-4x-2y$

Then the middle integral: $\int_0^{2-2x} (4-4x-2y)\,dy = \left[(4-4x)y - y^2\right]_0^{2-2x}$

$= (4-4x)(2-2x) - (2-2x)^2$

$= (2-2x)^2 = 4(1-2x+x^2)$

And finally the outer integral: $I = \int_0^1 4\cdot(1-2x+x^2)\,dx = 4\left[x - x^2 + \frac{x^3}{3}\right]_0^1 = \frac{4}{3}$

(2) I gives the volume of the tetrahedron bounded by the 3 coordinate planes ($x=0$, $y=0$, and $z=0$), and the plane with equation $z = 4-4x-2y$ (indeed, the trace of this plane on the plane $(z=0)$ is the line with equation $y = 2-2x$).