University of Utah
Math 1220, Fall 2007
Name: Solutions

Quiz # 4
Time: 15 minutes

Please try to carefully explain/justify the steps leading to your solutions.

Part 1: (10 points) Find the limit, if it exists, of $\frac{\ln(x^2)}{\sqrt{x}}$ as $x \longrightarrow +\infty$.

This is an indeterminate form " $\frac{\infty}{\infty}$ ", as $\ln(x^2) \xrightarrow[x\to+\infty]{} +\infty$ and $\sqrt{x} \xrightarrow[x\to+\infty]{} +\infty$.
We can therefore use L'Hôpital's rule and replace numerator and denominator by their derivative:

$$\lim_{x\to+\infty} \frac{\ln(x^2)}{\sqrt{x}} = \lim_{x\to+\infty} \frac{\frac{2x}{x^2}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to+\infty} \frac{4\sqrt{x}}{x} = \lim_{x\to+\infty} \frac{4}{\sqrt{x}} = 0$$

Thus: $\boxed{\frac{\ln(x^2)}{\sqrt{x}} \xrightarrow[x\to+\infty]{} 0}$

Part 2: (10 points) Find an antiderivative of the function $f(x) = \frac{1}{x^2-1}$ on the interval $(1, +\infty)$.

Note that $x^2-1 = (x+1)(x-1)$ has 2 real roots, ± 1.
Therefore we integrate by using partial fractions (and NOT $\tan^{-1}(x)$!).
We know that there exist constants A, B such that:

$$f(x) = \frac{1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

We find A and B by multiplying by $(x-1)(x+1)$: $1 = A(x-1) + B(x+1)$
and evaluating at $x=1$: $1 = 2B \longrightarrow B = \frac{1}{2}$
and at $x=-1$: $1 = -2A \longrightarrow A = -\frac{1}{2}$

Therefore: $\frac{1}{x^2-1} = -\frac{1}{2(x+1)} + \frac{1}{2(x-1)}$

And: $\int \frac{1}{x^2-1} = -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1|$ is an antiderivative of $f(x)$.
When $x > 1$, this is equal to $-\frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(x-1)$.