University of Utah
Math 1210, Spring 2008
Name: Solutions

Quiz # 3

Time: 15 minutes

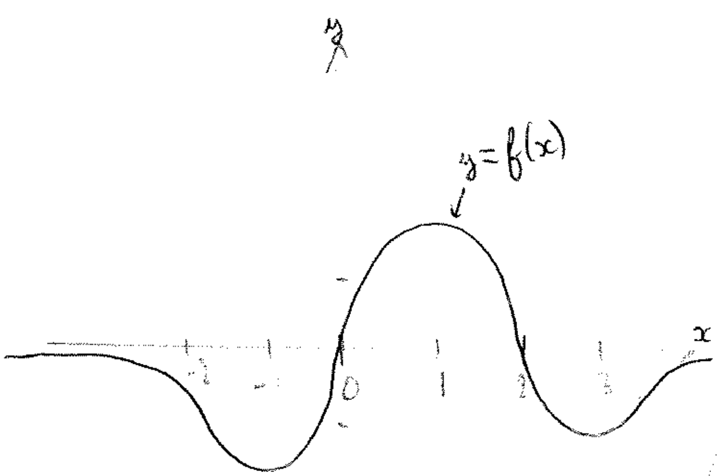
Show all work.

Part 1: (5 points) Consider the polynomial $P(x) = x^3 - 3x + 1$. Evaluate $P(-2)$, $P(0)$, $P(1)$ and $P(2)$. Prove that P has 3 real roots, and determine where these roots are situated with respect to -2, 0, 1 and 2.

$P(-2) = -1$, $P(0) = 1$, $P(1) = -1$, $P(2) = 3$. Thus P changes signs between -2 and 0, between 0 and 1, and between 1 and 2. Since P is a continuous function (it is a polynomial), we can use the Intermediate Value Theorem, which tells us that there exists c_1 between -2 and 0, c_2 between 0 and 1, and c_3 between 1 and 2 such that $P(c_1) = P(c_2) = P(c_3) = 0$.

(Algebra Note: P is a polynomial of degree 3, so it has at most 3 (real) roots.)

Part 2: (5 points). Consider the function f whose graph is sketched below. Determine for what values of x the derivative $f'(x)$ is zero, positive and negative, then sketch the graph of f'.



$f'(x)$ appears to be 0 for (approximately) $x = -1$, $x = 1$, and $x = 3$ (this is where the graph has a horizontal tangent). Moreover, $f'(x) < 0$ for $x \in (-\infty; -1) \cup (1; 3)$ (this is where f decreases) and $f'(x) > 0$ for $x \in (-1; 1) \cup (3; +\infty)$ (where f increases). Finally, $f'(x) \xrightarrow[x \to \pm\infty]{} 0$ because the graph of f has horizontal asymptotes at $\pm\infty$.

So the graph of f' will look something like this: