University of Utah
Math 2210, Fall 2008
Name: Solutions

Quiz # 1
Time: 10 minutes

Part 1: (8 points) Consider the function $f(x,y) = \frac{x^2+xy}{x^2+y^2}$. Determine whether or not $f(x,y)$ has a limit as $(x,y) \longrightarrow (0,0)$, and find this limit if it exists.

* If we restrict our attention to how f behaves on the axes:

$$\left.\begin{array}{l} f(x,0) = \frac{x^2}{x^2} = 1 \ (\text{for } x \neq 0) \text{ therefore: } \lim_{x\to 0} f(x,0) = 1 \\ f(0,y) = \frac{0}{y^2} = 0 \ (\text{for } y \neq 0) \text{ therefore: } \lim_{y \to 0} f(0,y) = 0 \end{array}\right\}$$

f has no limit at $(0,0)$ (different limits in different directions)

* You can also see this in polar coordinates: $\begin{cases} x = r.\cos\theta \\ y = r.\sin\theta \end{cases}$

for $r \neq 0$: $f(x,y) = \dfrac{r^2\cos^2\theta + r^2\cos\theta\sin\theta}{r^2} = \cos^2\theta + \cos\theta.\sin\theta$

so for fixed θ: $\lim_{r\to 0} f(x,y) = \cos^2\theta + \cos\theta.\sin\theta$

which yields different limits for different values of θ.

Part 2: (12 points) Consider the function $g(x,y) = xe^{x^2+y^2}$, and let S denote the graph of g. (a) Compute the partial derivatives of g. (b) Find the slope of the tangent to the curve of intersection of S with the plane $(x=1)$ at the point $(1,1,e^2)$.

(a) $\frac{\partial g}{\partial x}(x,y) = e^{x^2+y^2} + 2x.xe^{x^2+y^2} = (2x^2+1)e^{x^2+y^2}$ (product + chain rule)

$\frac{\partial g}{\partial y}(x,y) = 2y.xe^{x^2+y^2}$

(b) This is the geometric meaning of partial derivatives: the slope in question is $\frac{\partial g}{\partial y}(1,1) = 2e^2$