University of Utah
Math 2210, Fall 2008
Name: Solutions

Quiz # 2
Time: 10 minutes

A point $P(t)$ is moving in 3-space with position vector $\overrightarrow{OP(t)} = \vec{r}(t) = (\cos 2\pi t, \sin 2\pi t, e^t)$. Let C be the curve traced by the point $P(t)$.

1) Find the velocity and acceleration vectors and the speed of the point at time t. **(10 points)**

2) Find parametric equations for the tangent line to C at time $t = 1$. **(6 points)**

3)Sketch the curve C, first for $0 \leqslant t < \infty$ then $-\infty < t < \infty$. **(4 points)**

1) $\vec{v}(t) = \vec{r}'(t) = (-2\pi \sin 2\pi t, 2\pi \cos 2\pi t, e^t)$

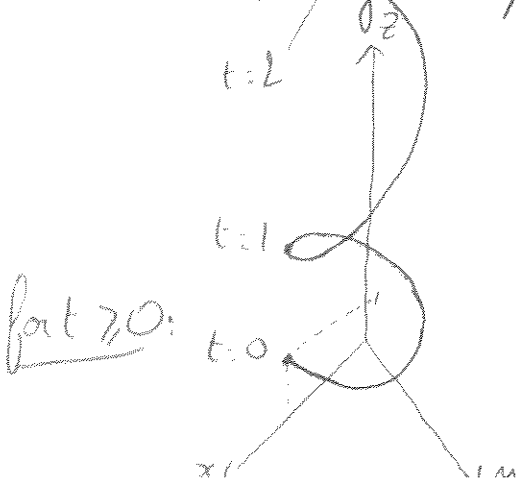
$\vec{a}(t) = \vec{r}''(t) = (-4\pi^2 \cos 2\pi t, -4\pi^2 \sin 2\pi t, e^t)$

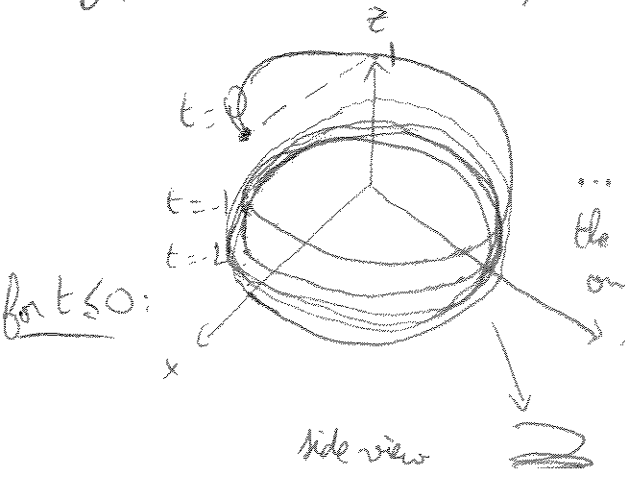
Speed: $v(t) = \|\vec{r}'(t)\| = \sqrt{(-2\pi \sin 2\pi t)^2 + (2\pi \cos 2\pi t)^2 + (e^t)^2} = \sqrt{4\pi^2(\sin^2 2\pi t + \cos^2 2\pi t) + e^{2t}} = \sqrt{4\pi^2 + e^{2t}}$

2) At time $t=1$: $\vec{r}(1) = (1, 0, e)$, the tangent line goes through the point $(1,0,e)$ with direction vector $\vec{v}(1) = (0, 2\pi, e)$, so parametric equations for this line are:

$$\begin{cases} x = 1 \ (+0.t) \\ y = (0 +) 2\pi . t \\ z = e + e . t \end{cases}$$

3) The coordinates $(\cos 2\pi t, \sin 2\pi t)$ describe the motion of a point going around a circle in the xy-plane, and the third coordinate is the height above the xy-plane at time t. The resulting curve is a helix where the height between two successive coils grows exponentially (for $t \geq 0$) or decreases exponentially (for $t \leq 0$)





... for $t \to -\infty$ the coils flatten onto the unit circle in the xy-plane