

# Solutions

## Quiz # 1

Time: 15 minutes

In parts 2 and 3, please try to carefully explain the steps leading to your solution.

**Part 1:** (6 points) State the two parts of the Fundamental Theorem of Calculus.

1) If  $f$  is a continuous function, then:

$F(x) = \int_a^x f(t) dt$  is an antiderivative of  $f$  (for any choice of  $a$ )

2) If  $f$  is continuous on the interval  $[a, b]$ , and if  $F$  is any antiderivative of  $f$ , then:  $\int_a^b f(t) dt = F(b) - F(a)$

**Part 2:** (8 points)

Evaluate the integral:

$$\int_0^1 \frac{x}{2-x^2} dx$$

Use the substitution:  $u = 2 - x^2$ ; then  $du = -2x dx$ , and  $u(0) = 2$   
 $u(1) = 1$

$$\text{Thus: } \int_0^1 \frac{x}{2-x^2} dx = -\frac{1}{2} \int_0^1 \frac{-2x}{2-x^2} dx$$

$$= -\frac{1}{2} \int_2^1 \frac{du}{u} = -\frac{1}{2} [\ln|u|]_2^1 = -\frac{1}{2} (\ln 1 - \ln 2) = \frac{\ln 2}{2}$$

Note: this is valid only because  $2 - x^2 \neq 0$  for  $x \in [0, 1]$ .

**Part 3:** (6 points)

Find the derivative of the function

$$f(x) = \ln(2 + \cos^2 x)$$

Use the chain rule:

$$f'(x) = \frac{1}{2 + \cos^2 x} \cdot (-2 \sin x \cos x)$$