University of Utah
Math 1090, Fall 2009
Name: Key

Midterm Exam # 2
Time: 50 minutes

No books, notes, calculators. Show all work. Check your answers.

**Problem 1:** (4 points) For the following matrices $A$ and $B$ find the products $AB$ and $BA$ if they exist:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 5 \\ 2 & 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 2 \end{pmatrix}$$

AB:

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 2 \end{pmatrix} = B$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 5 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 6 \\ 16 & 9 \\ 8 & 8 \end{pmatrix} = AB$$

BA: doesn't exist

(# columns of B $\neq$ # rows of A)

**Problem 2:** (8 points) Consider the function $f(x) = \frac{2-3x}{x+2}$, and denote $\Gamma$ its graph. (a) Find the domain of $f$ and the vertical asymptotes of $\Gamma$. (b) Find the horizontal asymptotes of $\Gamma$. (c) Determine the position of $\Gamma$ relative to its asymptotes (for horizontal ones: above/below, for vertical ones: does the graph go up or down to the left/right of the asymptote?) (d) Sketch the graph $\Gamma$.

(a) Domain: all $x \neq -2$

Vertical asymptote at $(x = -2)$

(b) Look at highest degree terms ("leading terms"): $\frac{-3x}{x} = -3$

so horizontal asymptote $(y = -3)$

(c) When $x$ is large and $>0$, $f(x) < 0$

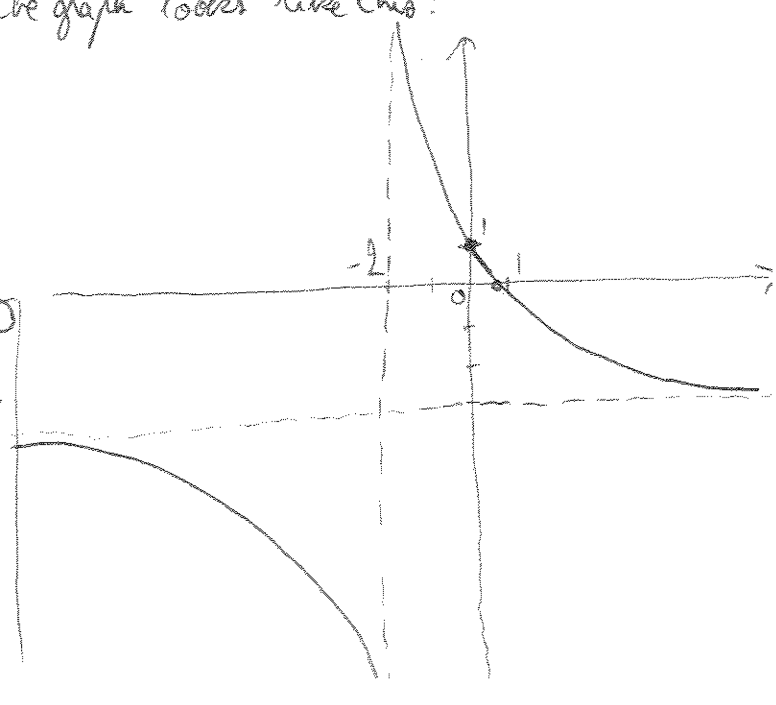
When $x$ is large and $<0$, $f(x) < 0$ also.

( $x$-intercept at $x = 2/3$, and:

$f(x) > 0$ for $-2 < x < 2/3$

$f(x) < 0$ for $x < -2$ )

(d) Using all of this (and the $y$-intercept $(0,1)$) the graph looks like this:



**Problem 3:** (5 points) If the profit function for a certain product is given by $P(x) = -4x^2 + 20x + 24$ (where $x$ is the number of units produced), find (a) the break-even point(s) and (b) the maximum profit and the number of units needed to achieve this maximum profit.

(a) Solve $P(x)=0$: $-4x^2+20x+24=0$ or $-x^2+5x+6=0$

Use quadratic formula (or sum/product of roots): $x=-1$ or $6$

Break-even point at $\boxed{x=6}$ (negative # units not defined).

(b) $P(x)$ is a quadratic function (parabola), it has a maximum at $x = -\frac{b}{2a} = -\frac{20}{-8} = \frac{5}{2} = 2.5$ Maximum profit is:

$P(2.5) = -25+50+24 = \boxed{49}$

**Problem 4:** (8 points) Solve the following system of equations by using the augmented matrix method:

$$\begin{cases} x + 2y - z = -3 \\ 2x + 3y + z = 4 \\ -x + y + 2z = 3 \end{cases}$$

The augmented matrix is:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 2 & 3 & 1 & 4 \\ -1 & 1 & 2 & 3 \end{array}\right)$$

Use row operations ("Gauss-Jordan elimination")

① 

② → replace by ② − 2×①

③ → replace by ③ + ①

New system

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & -1 & 3 & 10 \\ 0 & 3 & 1 & 0 \end{array}\right)$$

③ → replace by ③ + 3×②

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & -1 & 3 & 10 \\ 0 & 0 & 10 & 30 \end{array}\right) \to \boxed{z=3}$$

$\boxed{x=2}$ From this triangular system we

$\boxed{y=-1}$ get the solutions by backsubstituting:

Check in the original system:

$2 + 2\times(-1) + 3\times(-1) = -3$ ✓

$2\times 2 + 3\times(-1) + 3\times 1 = 4$ ✓

$-2 + (-1) + 2\times 3 = 3$ ✓