Bonus: * The previous analysis shows that f has exactly 4 roots. (the graph)
* The roots can be found by solving $-2x^4+4x^2-1$
Let $X=x^2$ and solve: $-2X^2+4X-1=0$
(Quadratic formula): $X=\frac{-4\pm\sqrt{8}}{-4}=1\pm\frac{\sqrt{2}}{2}$ (both >0!)
Then: $x=\pm\sqrt{1\pm\frac{\sqrt{2}}{2}}$.

University of Utah
Math 1210, Spring 2008
Name: Solutions

Midterm Exam # 2
Time: 50 minutes

No books, notes, calculators. Show all work.

Part 1: (16 points) Consider the function $f(x)=-2x^4+4x^2-1$. (a) Find the limits of $f(x)$ as $x\to\pm\infty$. (b) Evaluate the derivative $f'(x)$; find the critical points of f and determine on which intervals f is increasing or decreasing. (c) Evaluate the second derivative $f''(x)$; find the inflection points of f and determine on which intervals f is concave up or down. (d) Find the local maxima and minima of f. (e) Sketch the graph of f, including all special points. (f) Does f have a global maximum? a global minimum? (g) Sketch the graph of f'.

Bonus: (2 points) How many roots (x-intercepts) does f have? (justify your answer). Find these roots.

(a) $\lim_{x\to\pm\infty} f(x)=-\infty$ (leading term $-2x^4$).

(b) $f'(x)=-8x^3+8x=-8x(x-1)(x+1)$
The critical points of f are therefore $-1, 0, 1$, and the sign of f' is given by the table:

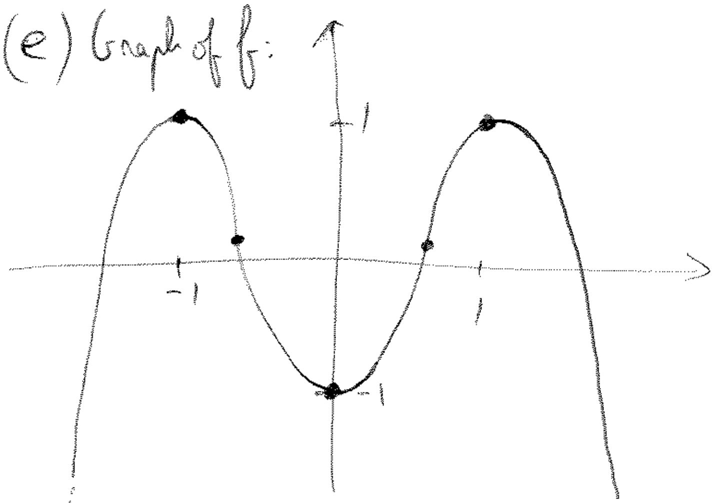
x	$-\infty$		-1		0		1		$+\infty$
f'		$+$	0	$-$	0	$+$	0	$-$	
f	$-\infty$	↗	1	↘	-1	↗	1	↘	$-\infty$

f is increasing on $(-\infty;-1)\cup(0;1)$
decreasing on $(-1;0)\cup(1;+\infty)$.

(c) $f''(x)=-24x^2+8=-24\left(x-\frac{1}{\sqrt{3}}\right)\left(x+\frac{1}{\sqrt{3}}\right)$
Therefore $f''(x)>0$ for $x\in\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ (f concave up)
$f''(x)<0$ for $x\in\left(-\infty;-\frac{1}{\sqrt{3}}\right)\cup\left(\frac{1}{\sqrt{3}};+\infty\right)$ (f concave down)
and $\left(-\frac{1}{\sqrt{3}},\frac{1}{9}\right)$ $\left(\frac{1}{\sqrt{3}},\frac{1}{9}\right)$ are the inflection points of f
($f''(x)=0$ for $x=\pm\frac{1}{\sqrt{3}}$ and f'' changes signs at these points).

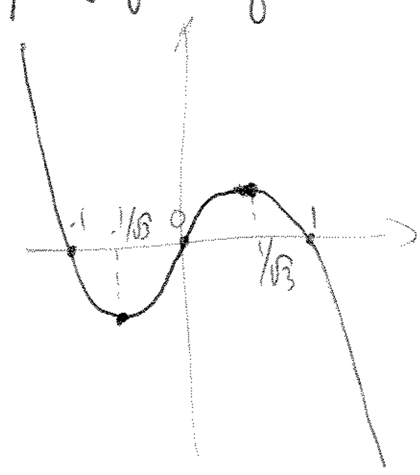
(d) Therefore f has local maxima of 1 at -1 and 1, and a local minimum of -1 at 0.

(e) Graph of f:



(f) f has a global maximum of 1, attained at -1 and 1, but no global minimum.

(g) Graph of f': $f'(x)=-8x^3+8x$



Part 2: (5 points) Let $g(x) = x\sin(5x) + \cos(x^2)$ and $h(x) = \frac{x\sin(5x)+\cos(x^2)}{x+1}$. (a) Evaluate the derivatives of g and h. (b) Find an equation for the tangent to the graph of h at the point $(0, 1)$.

(a) * $g'(x) = 1.\sin(5x) + x.5\cos(5x) - 2x\sin(x^2)$ (product and Chain Rule).

* Use the Quotient Rule; we just evaluated the derivative of the numerator:

$$h'(x) = \frac{(x+1)[\sin(5x) + 5x\cos(5x) - 2x\sin(x^2)] - x\sin(5x) - \cos(x^2)}{(x+1)^2}$$

(b) This looks scary, but we only need to evaluate $h'(x)$ at $x=0$, knowing that $\sin(0)=0$ and $\cos(0)=1$:

$$h'(0) = \frac{1.[0+0+0] - 0 - 1}{1} = -1.$$

This is the slope of the tangent to the graph of h at the point $(0,1)$; an equation of this tangent is therefore: $y = -x+1$