University of Utah
Math 1090, Fall 2009
Name: Key

Midterm Exam # 1

Time: 50 minutes

No books, notes, calculators. Show all work. Check your answers.

Problem 1: (3 points) Simplify the expression $\frac{\sqrt{28x^{-1}y^3}}{\sqrt{7x^{-3}y^5}}$.

$$\frac{\sqrt{28x^{-1}y^3}}{\sqrt{7x^{-3}y^5}} = \sqrt{\frac{28x^{-1}y^3}{7x^{-3}y^5}} = \sqrt{4x^2y^{-2}} = 2|x|\cdot|y|^{-1} = 2\frac{|x|}{|y|}$$

Problem 2: (4 points) Consider the points $P=(1,2)$ and $Q=(3,4)$. (a) Find an equation for the line L_1 joining P and Q. (b) Find an equation for the line L_2 perpendicular to L_1 through P. (a) The slope of L_1 is $\frac{4-2}{3-1}=1$, and the point-slope equation is: $y-2=1.(x-1)$ or $\boxed{y=x+1}$

(b) L_2 being perpendicular to L_1 has slope $-\frac{1}{1}=-1$, so the point-slope equation is: $y-2=-1(x-1)$ or $\boxed{y=-x+3}$

Problem 3: (7 points) Solve the following system of equations:

$$\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}\begin{cases} x-y+z=2 \\ 2x-y-z=-10 \\ -3x+y-2z=-3 \end{cases}$$

Use left-to-right elimination:

Step 1: Replace equation 2 by 2 - 2 1 and 3 by 3 + 3 1.

The new system is:

$$\begin{matrix} 1 \\ 2' \\ 3' \end{matrix}\begin{cases} x-y+z=2 \\ \quad y-3z=-14 \\ \quad -2y+z=3 \end{cases}$$

Step 2: Replace 3' by 3' + 2 2', getting: $-5z=-25$ or $\boxed{z=5}$

Backsubstitution then gives $\boxed{y=1}$ and $\boxed{x=-2}$.

We check these solutions by plugging them into the 3 original equations. ✓

Problem 4: (4 points) A manufacturer makes and sells calculators for \$25 each. Fixed costs are \$1800 and variable costs are \$5 per unit. (a) Write the total cost, revenue and profit functions. (b) Find the break-even point.

$x = \#\left\{\begin{array}{l}\text{units made}\\ \text{and sold}\end{array}\right\}$

(a) Cost: $C(x) = 1800 + 5x$

Revenue: $R(x) = 25x$

Profit: $P(x) = 25x - (1800 + 5x) = 20x - 1800$

(b) Break-even point: solve $P(x) = 0$: $20x = 1800$ so $\boxed{x = 90}$

Problem 5: (7 points) A computer retail store will buy 600 units at \$100 each, but only 400 units if they are \$200 each. The wholesaler is willing to supply 700 units at \$200, but only 300 at \$100 each. Assuming that the supply and demand functions are linear: (a) Find the supply and demand functions (using p for price and q for quantity) (b) Find the market equilibrium point.

(a) ~~Supply~~ Demand: assumed linear, we can write it as $p = aq + b$

We find a and b by plugging in the specified values:

$$\begin{cases} ① \ 100 = a \cdot 600 + b \\ ② \ 200 = a \cdot 400 + b \end{cases}$$

Subtract ② from ①: $200a = -100$ so $a = -\frac{1}{2}$

and $b = 200 - 400a = 400$ (from ②)

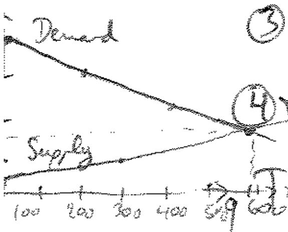
Therefore the demand function is: $\boxed{p = -\frac{1}{2}q + 400}$

Supply: Write as $p = cq + d$ and find c and d by plugging in values:

$$\begin{cases} ③ \ 200 = c\,700 + d \\ ④ \ 100 = c\,300 + d \end{cases}$$

Subtract ④ from ③: $400c = 100$ so $c = \frac{1}{4}$

and (from ④) $d = 100 - 300c = 25$



Therefore the supply function is: $\boxed{p = \frac{1}{4}q + 25}$

(b) At market equilibrium, the p's and q's for supply and demand are equal (it's the intersection point of the 2 lines) so:

$p = -\frac{1}{2}q + 400 = \frac{1}{4}q + 25$ gives: $\frac{3}{4}q = 400 - 25 = 375$

so $\boxed{q = 500}$ and $\boxed{p = -\frac{1}{2}q + 400 = 150}$