Step 2:

If **P'** is on the positive x-axis along the line $\overline{O'R'}$ as show in Figure 11, the axis of rotation is that obtained from equation (1) but rotated by $\omega = \mathbf{f}\left(\frac{|\overrightarrow{O'P'}|}{|\overline{O'R'}|}\right)$ degrees about the y-axis. Namely,

$$\vec{a}\,(x,y,z) = [-\sin(\tau')\ \cos(\tau')\ 0]\cdot\begin{bmatrix}\cos f(\omega) & 0 & -\sin f(\omega)\\ 0 & 1 & 0\\ \sin f(\omega) & 0 & \cos f(\omega)\end{bmatrix} \qquad (2)$$

where $|\overrightarrow{O'P'}|$ and $|\overline{O'R'}|$ are the length of the (2-D) vector $\overrightarrow{O'P'}$ and the line $\overline{O'R'}$ respectively, and **f**(x) can be any monotonically-increasing function with conditions:

$$f(x) = \begin{cases} 0° & \text{if } x \le 0 \\ 90° & \text{if } x \ge 1 \end{cases}$$

The function **f**(x) describes how the hemisphere is distorted into the flat disk. The Virtual Sphere controller in the experiments used **f**(x) = x, with the above constraints. Note that if $|\overrightarrow{O'P'}| = 0$, equation (2) is the same as equation (1). If $|\overrightarrow{O'P'}| = |\overline{O'R'}|$, then the axis of rotation is on the y-z plane.

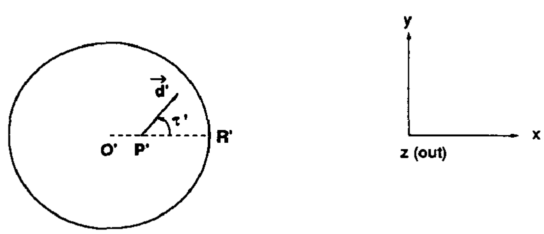

Figure 11. Movement of the 2-D device where **P'** is on $\overline{O'R'}$.

Step 3:

In the general case (see Figure 12),

$\overrightarrow{O'P'}$ makes angle θ' with the x-axis, and
$\vec{d}$ " " θ'+τ' " ".

Since Figure 12 is just Figure 11 rotated by θ' degrees about the z-axis, the axis of rotation is that obtained from equation (2) excepted rotated by θ' degrees about z. Namely,

$$\vec{a}\,(x,y,z) = [-\sin(\tau')\ \cos(\tau')\ 0]\cdot\begin{bmatrix}\cos f(\omega) & 0 & -\sin f(\omega)\\ 0 & 1 & 0\\ \sin f(\omega) & 0 & \cos f(\omega)\end{bmatrix}\cdot\begin{bmatrix}\cos\theta' & \sin\theta' & 0\\ -\sin\theta' & \cos\theta' & 0\\ 0 & 0 & 1\end{bmatrix} \qquad (3)$$

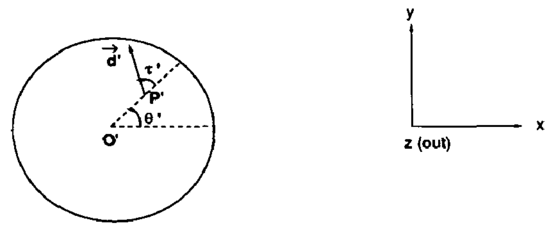

Figure 12. Movement of the 2-D device where **P'** is arbitrarily located.

Once the axis of rotation is obtained from equation (3), the rotation matrix **R** can be computed by:

$$\mathbf{R}_{\vec{a}}\,(\varphi) = \begin{bmatrix} ta_x^2+c & ta_xa_y+sa_z & ta_xa_z-sa_y \\ ta_xa_y-sa_z & ta_y^2+c & ta_ya_z+sa_x \\ ta_xa_z+sa_y & ta_ya_z-sa_x & ta_z^2+c \end{bmatrix} \qquad (4)$$

where a_x, a_y and a_z are the components of $\vec{a}$, s = sinφ, c = cosφ, and t = 1-cosφ, and φ is the amount of rotation about $\vec{a}$ [7].

The angle of rotation φ can simply be the distance of cursor movement times a suitable scaling factor. However, we decided to model the rolling of the sphere more precisely. We scaled the amount of rotation such that:

1) a full sweep of the mouse across the circle (passing through **O'**) produces 180 degrees of rotation;
2) a full circle around the edge (or outside) the circle produces 360 degrees of rotation.

The following formula for φ in degrees (obtained empirically) was used in the experiment, and provides a good approximation to the two desirable properties described above:

$$\varphi = 90° * \frac{|\vec{d}|}{|\overline{O'R'}|}\left\{1 - \left(1 - \frac{0.2}{\pi}\right)\frac{\omega}{90°}\left(1 - |\cos\tau'|\right)\right\} \qquad (5)$$