Math 2280-1 Notes of August 26, 2022

1.3 Slope Fields and Solution Curves

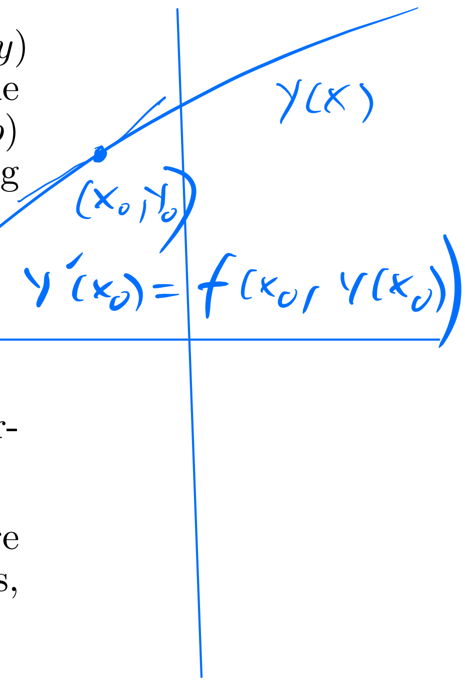
- Continue to consider

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$$

Theorem 1. (p.22) Suppose that both the function $f(x, y)$ and its partial derivative $D_y f(x, y)$ are continuous on some rectangle R int the xy-plane that contains the point (a, b) in its interior. Them for some open Interval I containing the point a, the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y), \qquad y(a) = b$$

has one and only one solution that is defined on the interval I.



- We won't worry about proving the theorem. There are courses on DEs that focus on proofs of results like this, but not this course.

- Usually existence and uniqueness are not a problem.

- Today we'll have fun drawing solution curves.
- There are many computer packages available for this purpose.
- I was hoping to show you some particular software, but I can't get it to work on my Ipad.
- Instead I found one online
- check out

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https://aeb019.hosted.uark.edu/dfield.html
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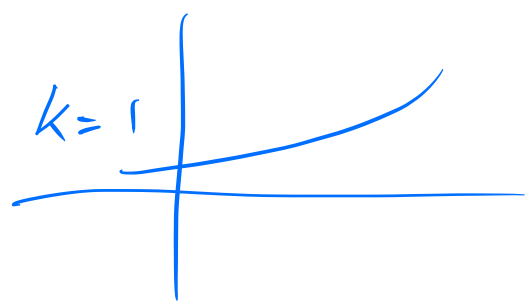
- Recall definition of **slope field**.
- Associate a small line segment with slope $f(x, y)$ with every point (x,y) in the plane.



$$P' = kP$$

$c_1 e^{kt}$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP(M - P)$$

(logistic equation)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k(P - m)(M - P)$$

(logistic equation)

$$y' = y^2$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP^p \qquad y' = y^p$$

$$y' = -\cancel{\frac{x}{y}} \quad -\frac{y}{x}$$

$$x^2 + y^2 = r^2$$

$$2x + 2yy' = 0$$

$$y' = \frac{-x}{y}$$

$$y' = x - y$$

$$y' = y^p$$

$$\frac{dy}{dt} = y^p$$

$$y^{-p}\,dy = dt$$

$$\frac{y^{-p+1}}{-p+1} = t + c_1$$

$$y^{1-p} = (1-p)(t+c_1)$$

$$y = \big((1-p)(t+c_1)\big)^{\frac{1}{1-p}}$$

$$v' = -g + kv$$

(air resistance!)

$$v' = -g + kv^2$$

(air resistance!)

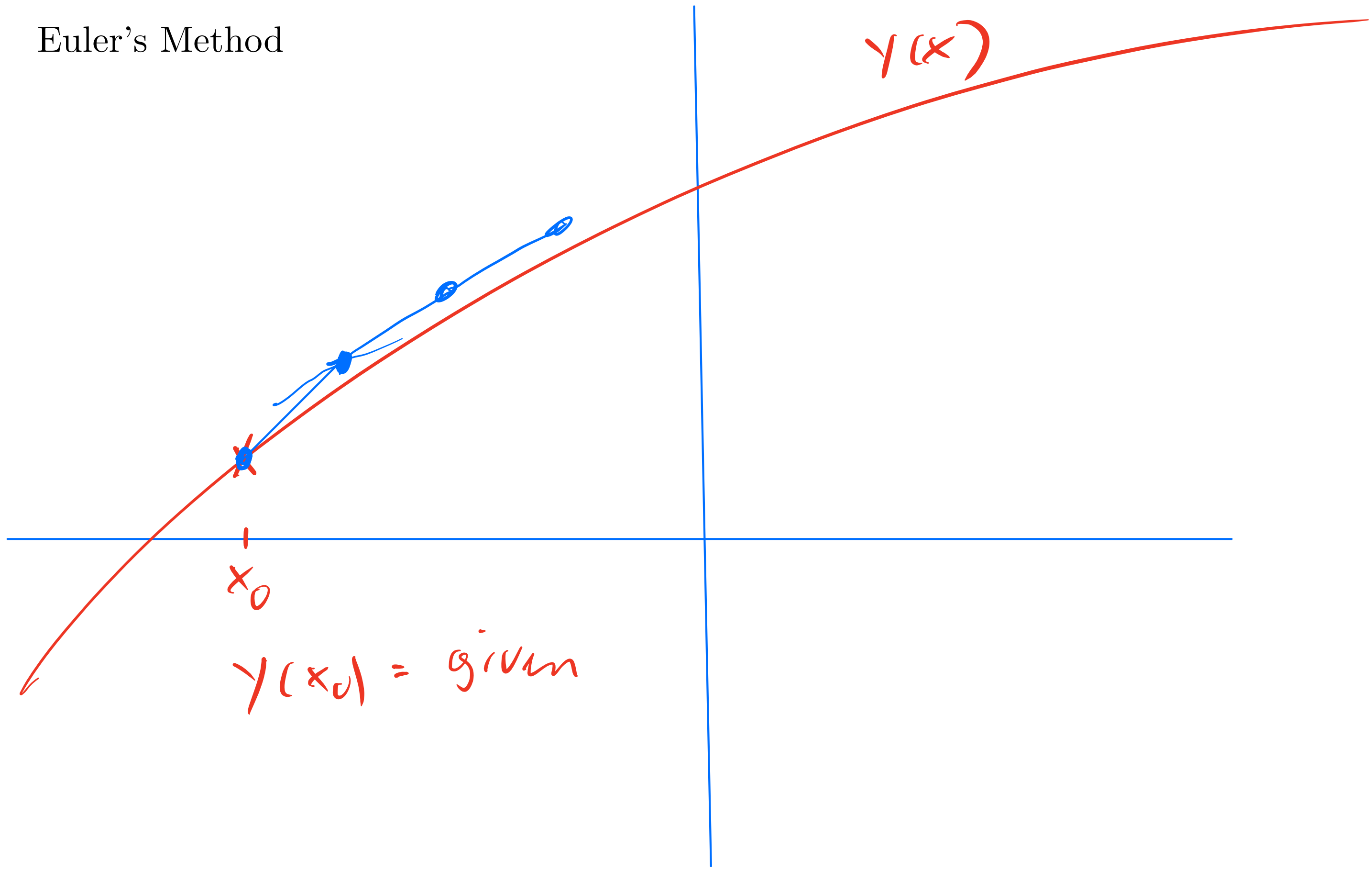
$$y' = 1/x$$

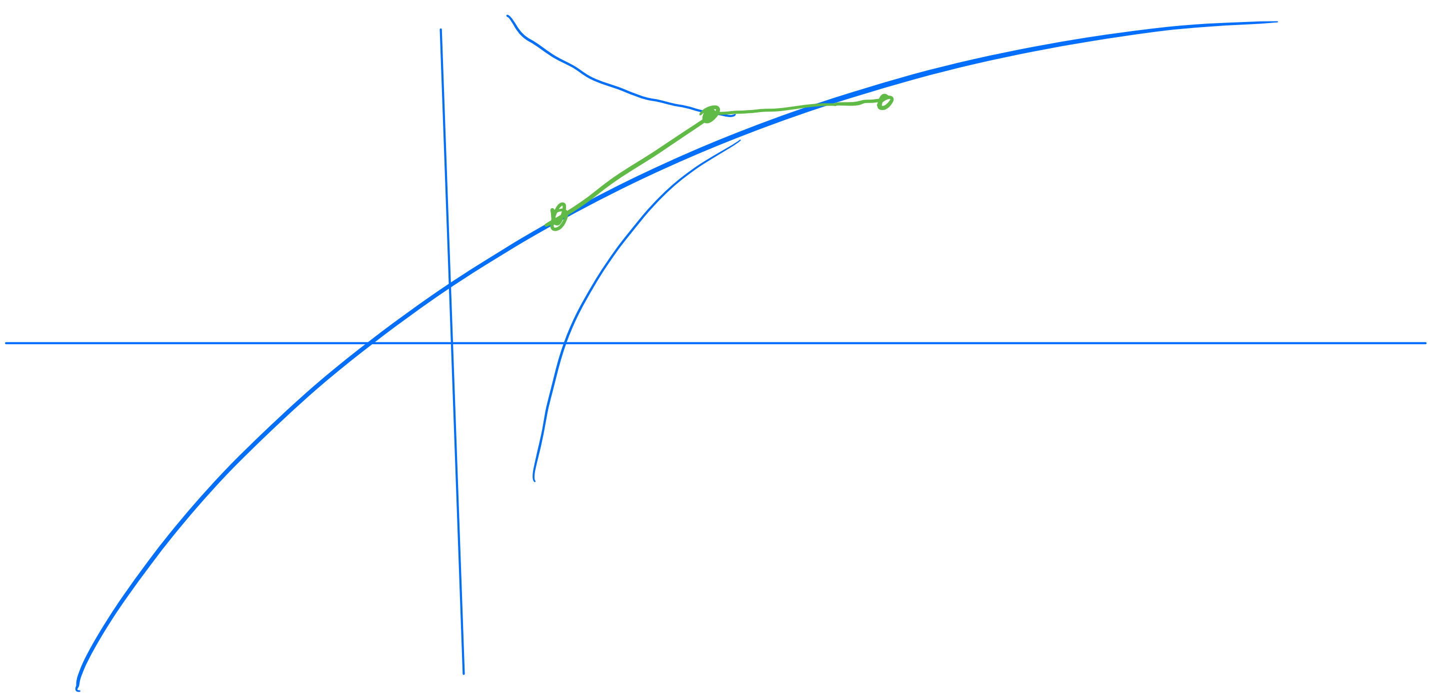
(Example 5)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP(M-P)$$

(logistic equation)

Euler's Method





$$y' = \lambda(y - \sin x) + \cos x$$