Math 2280-1 Notes of August 26, 2022

1.3 Slope Fields and Solution Curves

- Continue to consider

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$$

Theorem 1. (p.22) Suppose that both the function $f(x, y)$ and its partial derivative $D_y f(x, y)$ are continuous on some rectangle R int the xy-plane that contains the point (a, b) in its interior. Them for some open Interval I containing the point a, the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y), \qquad y(a) = b$$

has one and only one solution that is defined on the interval I.

- We won't worry about proving the theorem. There are courses on DEs that focus on proofs of results like this, but not this course.
- Usually existence and uniqueness are not a problem.

- Today we'll have fun drawing solution curves.
- There are many computer packages available for this purpose.
- I was hoping to show you some particular software, but I can't get it to work on my iPad.
- Instead I found one online
- check out

`https://aeb019.hosted.uark.edu/dfield.html`

- Recall definition of **slope field**.
- Associate a small line segment with slope $f(x,y)$ with every point (x,y) in the plane.

$$P' = kP$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP(M-P)$$

(logistic equation)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k(P-m)(M-P)$$

(logistic equation)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP^p$$

$$y' = -\frac{x}{y}$$

$$y' = x - y$$

$$v' = -g + kv$$

(air resistance!)

$$v' = -g + kv^2$$

(air resistance!)

$$y' = 1/x$$

(Example 5)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP(M - P)$$

(logistic equation)

Euler's Method

$$y' = \lambda(y - \sin x) + \cos x$$