

1.1 Differential Equations and Mathematical Models

- A differential equation is an equation that involves a function and some of its derivatives.
- DEs are important because they describe natural processes
- **Examples:**

$$\frac{dx}{dt} = x^2 + t^2$$

$$\frac{d^2y}{dt} = \frac{C}{y^2}$$

Short List of Possible Questions

- Given a physical situation, find a differential equation that describes it.
- Given a differential equation, find a solution, either exactly, or approximately.
- Interpret a solution.
- Find families of solutions and describe their relationships and properties.
- Examine the dependence of a solution on certain parameters.

Families of Solutions

- Example 2, page 2.

$$y(x) = Ce^{x^2}$$

Example 3, Newton's Law of Cooling

The rate of change of the temperature of an object is proportional to the difference of the ambient temperature and the temperature of the object.

- **Toricelli's law:** Water drains at the speed of an object falling from a height that equals the depth of the water. This implies that the rate at which the water drains is proportional to the square root of the depth.

- The **growth of a population** may be proportional to the size of the population. This gives rise to **exponential growth**.
- Note that we talk about the derivative of an integer valued function which strictly speaking does not make sense.

Some Terminology

Illustrate with

$$\frac{dy}{dx} = y^2$$

- solving a differential equation
- general solution
- particular solution
- initial condition
- initial value problem
- parameter
- order of a differential equation
- ordinary differential equation
- partial differential equation

- Example 10: Solve the initial value problem

$$y' = y^2, \quad y(1) = 2.$$

Suppose P denotes a population.

Discuss qualitative differences between the solutions of

$$P' = kP \quad \text{and} \quad P' = kP^2$$