Math 2280-1   Notes of August 22, 2022

I am not the instructor

- Peter Alfeld, pa@math.utah.edu
- will cover the class for the first week
- The instructor is Quentin Posva, posva@math.utah.edu
- You should have heard from him via Canvas

- He asked me to mention the following points to you:

1. All the information is on Canvas. You can write Quentin by email or via Canvas if you have questions.
2. Weekly assignments, that will be part of the final grade. To be written on paper, individually, and stapled. Quentin will collect them each Monday at the beginning of the class.
3. There will be a midterm, towards the middle of the semester.
4. There will a computer project, towards the 2/3 of the semester, that will be part of the final grade. Working in groups is encouraged.

- I will be happy to answer your questions and meet with you during this week. Talk with me after class or email me and we can set up an appointment.
- These notes will be on

`http://www.math.utah.edu/~pa/2280`

- We will complete them together in class and then the annotated versions will also be online.
- Today: introduction, and during the rest of the week we will cover the first three sections of the textbook

- A **differential equations** (DE) is an equation that involves a function and some of its derivatives.
- You have already seen some examples of DEs in Calculus.
- Some Examples:

$y' = 0$ $\qquad y(x) = C_1$

$y' = 1$

$y(x) = x + C_1$

$y' = x$

$y(x) = \frac{x^2}{2} + C_1$

$y = e^{x + \hat{C}_1}$

$y' = e^{x + \hat{C}_1}$

$= e^{\hat{C}_1} e^x$

$= C_1 e^x$

$= y$

$y' = y$

$y(x) = e^x$

$y' = e^x$

$y' = ky$

$y(x) = a^x = \left(e^{\ln a}\right)^x = e^{x \ln a}$ $\qquad \ln a = k$

$y' = e^x = y$

$y' = \ln a\, e^{x \ln a} = k a^x$

$= k\left(e^{\ln a}\right)^x$

$= k e^{kx}$

$y' = f(x)$

$y(x) = \int_a^x f(t)\,dt$

$y(x) = e^{kx}$

$y'(x) = k e^{kx} = ky$

$y(x) = C_1 e^{kx}$

$y'(x) = C_1 k e^{kx}$

$= ky$

- Differential Equations form a huge subject. They are so important because they describe physical ("real life") processes.

- To illustrate the point I list our courses focused on DEs: 2250, **2280**, 3140, 3150, 5410, 5420, 5440, 5620, 6410, 6420, 6430, 6440, 6620, 6630, 6845, 6850, 7845.

- We start with a simple and familiar physical example. See also Example 3, page 14, textbook

- If we release an object near the surface of earth it will fall at an increasing speed. We can measure that the speed increases by 32 feet per second every second. We say it increases by 32 feet per second squared.

- This is not realistic since as the speed increases so does air resistance, and also because gravity increases as we get closer to earth.

- But let's ignore these issues for the time being.

- Let's agree the positive direction is up.

- Instead of 32 ft/sec^2 let's use g

- The more you go on in math the less you will see numbers, and the more you'll use variables.

- Consider these three related functions:

$h(t)$ height (above the surface of the earth) at time t (measured in seconds).

$v(t)$ vertical velocity, positive if up, negative if down.

$a(t)$ vertical acceleration.

These three functions are interrelated!

$a = v'$

$v = h'$

$$a(t) = -32$$

$$v(t) = -32t + c_1 \qquad v(0) = c_1 = v_0$$

$$= -32t + v_0$$

$$h(t) = -16t^2 + v_0 t \qquad + c_1 + c_2$$

$$= -16t^2 + v_0 t + h_0$$

- Thus we get

$$h(t) = -\frac{gt^2}{2} + v_0 t + h_0$$
$$v(t) = h'(t) = -gt + v_0$$
$$a(t) = h''(t) = v'(t) = -g$$

where h_0 is the initial altitude and v_0 is the initial velocity. Note that we started with the acceleration and then computed velocity and height.

Given these equations we can now ask questions like

- How long does an object stay in the air? ✓ 5s
- What maximum height does it reach? 144'
- At what speed will it hit the ground? -96 ft/s
- The first problem asks for a time. The other two can be answered by first computing the time at which something happens.
- Let's illustrate this with the specific example

$$h(t) = -16t^2 + 64t + 80$$

$$h(t) = -16(t^2 - 4t - 5) = 0$$
$$= -16(t-5)(t+1) = 0$$
$$t = -1 \quad \text{or} \quad t = 5$$

$$v(t) = h'(t) = -32t + 64 = 0 \Rightarrow t = 2$$
$$h(t) = -16 \cdot 4 + 64 \cdot 2 + 80 = 144 \text{ feet}$$
$$v(5) = -32 \cdot 5 + 64 = -96 f/s$$

$$a = -g$$

$$v = -gt + v_0$$

$$h = -\frac{g}{2}t^2 + v_0 t + h_0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$h(t) = 0$?

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2gh_0}}{-g}$$

$$v(t) = -gt + v_0 = 0$$

$$t = \frac{v_0}{g} \qquad \left[\frac{f/s}{f/s^2} = s \right]$$

$$h(t) = -\frac{g}{2}\frac{v_0^2}{g^2} + v_0 \frac{v_0}{g} + h_0$$

$$= \frac{v_0^2}{2g} + h_0$$

$$\frac{f^2/s^2}{f/s^2} = f$$

Of course, we can just as well solve these problems in general for the case that

$$h(t) = -\frac{gt^2}{2} + v_0 t + h_0$$