

-1- (Matrix Multiplication.)

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 1 & 3 \end{bmatrix} = AA^T \quad \text{and} \quad \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 5 & 5 \\ 4 & 5 & 10 \end{bmatrix} = A^T A$$

-2- (Rank.)

$$AB = \begin{bmatrix} 1 \\ 3 \end{bmatrix} [2 \ 1] + \begin{bmatrix} 2 \\ 4 \end{bmatrix} [1 \ 2] = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 10 & 11 \end{bmatrix}.$$

$$\begin{bmatrix} 4 & 5 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 10 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 10 & 11 \end{bmatrix}.$$

-3- (LU-factorization.)

$$L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 1 & 2 \\ 3 & 4 \end{bmatrix} = U = A$$

-4- (Inverse Matrix.) Compute the inverse of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix} = A^{-1}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

-5- (Block Matrices.)

$$A^{-1} = \begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ \mathbf{0} & A_{22}^{-1} \end{bmatrix}$$

-6- (True or False.) For each of the following statements indicate whether they are true (T) or false (F). Assume A is an $m \times n$ matrix. You do not need to give reasons for your answer.

1. F 2. T 3. T 4. F 5. T 6. F 7. T 8. T 9. T 10. T
F T T F T F T T T T