Math 2270-1

Matrix Multiplication

$$B \quad n \times p$$

$$\begin{bmatrix} b_{11} & \dots & b_{1j} & \dots & b_{1p} \\ \vdots & & \vdots & & \vdots \\ b_{i1} & \dots & b_{ij} & \dots & b_{ip} \\ \vdots & & \vdots & & \vdots \\ b_{n1} & \dots & b_{nj} & \dots & b_{np} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{m1} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix} \qquad \begin{bmatrix} & & & & \\ & & \vdots & & \\ & \dots & c_{ij} & \dots & \\ & & \vdots & & \\ & & & & \end{bmatrix}$$

$$A \quad m \times n \qquad\qquad C = AB \quad m \times p$$

It is evident from this picture that

- the $i-j$ entry of C is the (dot) product of the i-th row of A and the j-th column of B,
- the j-th column of C is the product of A and the j-th column of B,
- the i-th row of C is the product of the i-th row of A and B.

- Following is an elaboration of these views. But first, here is a clean copy of the same picture:

$$
\begin{array}{cc}
 & B \quad n \times p \\
 & \begin{bmatrix} b_{11} & \dots & b_{1j} & \dots & b_{1p} \\ \vdots & & \vdots & & \vdots \\ b_{i1} & \dots & b_{ij} & \dots & b_{ip} \\ \vdots & & \vdots & & \vdots \\ b_{n1} & \dots & b_{nj} & \dots & b_{np} \end{bmatrix} \\
\begin{bmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{m1} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix} & \begin{bmatrix} & & & & \\ & & \vdots & & \\ & \dots & c_{ij} & \dots & \\ & & \vdots & & \\ & & & & \end{bmatrix} \\
A \quad m \times n & C = AB \quad m \times p
\end{array}
$$