Write your name here:

Math 2270-1 — Fall 2019 — Exam 4

1 **2** **3** **4** **5** **Total**

Instructions

1. **This exam is closed books and notes. Do not use a calculator or another electronic device. Do not use scratch paper.**

2. Use these sheets to record your work and your results. Use the space provided, and the back of these pages if necessary. **Show all work.** Unless it's obvious, indicate the problem each piece of work corresponds to, and for each problem indicate where to find the corresponding work.

4. To avoid distraction and disruption **I am unable to answer questions during the exam.** If you believe there is something wrong with a problem state so, and if you are right you will receive generous credit. I will also be unable to discuss individual problems and grading issues with you after you are done while the exam is still in progress.

5. If you are done before the allotted time is up I recommend strongly that you stay and use the remaining time to **check your answers.**

6. When you are done hand in your exam, pick up an answer sheet, and leave the room. Do not return to your seat.

7. All questions have equal weight.

8. Clearly indicate (for example, by circling or boxing) your final answers.

-1- (Gram Schmidt Process.) Using the Gram-Schmidt Process, find an orthonormal basis of the space

$$W = \text{span}\{\mathbf{x}_1, \mathbf{x}_2\} \quad \text{where} \quad \mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

-2- (**Inner Product Spaces.**) Suppose W is the inner product space of polynomials of degree up to 20, with the inner product

$$< p, q >= \int_0^1 p(x)q(x)\mathrm{d}x.$$

Compute the norm of $p(x) = x^3$.

-3- **(Inner Products.)** Again, let W be the inner product space of polynomials of degree up to 20. Suppose you define

$$< p, q >= p(1)q(1) + p(2)q(2) + p(3)q(3) + p(4)q(4).$$

Is $< \cdot, \cdot >$ and inner product on W? Why, or why not?

-4- (Least Squares.)   Suppose you want to solve the least squares problem

$$\|A\mathbf{x} - \mathbf{b}\| = \min$$

where A is $m \times n$ with $m > n$ and $\mathbf{b} \in \mathbb{R}^m$ is a given vector. Describe and derive the normal equations giving the solution of this problem.

-5- (True or False.) Mark the following statements as true or false by circling **F** or **T**, respectively. You need not give reasons for your answers. Unless stated otherwise, all linear spaces are inner product spaces and finite dimensional.

1. T F All orthogonal matrices are symmetric.

2. T F The determinant of an orthogonal matrix is real.

3. T F The zero vector in an inner product space is orthogonal to all vectors in that space.

4. T F Suppose W is a subspace of a finite dimensional inner product space and $W^\perp$ is its orthogonal complement. Then the intersection of W and $W^\perp$ is empty.

5. T F The orthogonal projection of the zero vector into a subspace of an inner product space is the zero vector.

6. T F Every inner product space has an orthogonal basis.

7. T F Suppose W is a subspace of an inner product space V, and $W^\perp$ is its orthogonal complement. Then the dimension of V equals the sum of the dimensions of W and $W^\perp$.

8. T F The projection of a vector $\mathbf{y}$ into a subspace W is unique.

9. T F The solution of the normal equations $A^T A \mathbf{x} = A^T \mathbf{b}$ is unique.

10. T F For any two vectors $\mathbf{x}$ and $\mathbf{y}$ in an inner product space it is true that

$$\|\mathbf{x} + \mathbf{y}\| \le \|\mathbf{x}\| + \|\mathbf{y}\|.$$