Write your name here:

Math 2270-1 — Fall 2019 — October 2, 2019 — Exam 2

1	2	3	4	5	6	7	8	Total

Instructions

1. **This exam is closed books and notes. Do not use a calculator or other electronic devices. Do not use scratch paper.**
2. Use these sheets to record your work and your results. Use the space provided, and the back of these pages if necessary. **Show all work.** Unless it's obvious, indicate the problem each piece of work corresponds to, and for each problem indicate where to find the corresponding work.
4. To avoid distraction and disruption **I am unable to answer questions during the exam.** If you believe there is something wrong with a problem state so, and if you are right you will receive generous credit. I will also be unable to discuss individual problems and grading issues with you after you are done while the exam is still in progress.
5. If you are done before the allotted time is up I recommend strongly that you stay and use the remaining time to **check your answers.**
6. When you are done hand in your exam, pick up an answer sheet, and leave the room. Do not return to your seat.
7. All questions have equal weight.
8. Clearly indicate (for example, by circling or boxing) your final answers.

-1- (Matrix Multiplication.) In one sentence, state why we multiply matrices the way we do.

-2- (Matrix Multiplication.) Let

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Compute AA^T and A^TA.

-3- (Block Matrices.) Suppose A is the invertible upper triangular block matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ \mathbf{0} & A_{22} \end{bmatrix}$$

where A_{11} is $p \times p$ and A_{22} is $q \times q$. (Of course, A_{12} is $p \times q$ and $\mathbf{0}$ is the $q \times p$ zero matrix.) Compute the inverse block matrix

$$A^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

where B_{11} is $p \times p$, B_{12} is $p \times q$, B_{21} is $q \times p$, and B_{22} is $q \times q$. Express the blocks of A^{-1} in terms of the blocks of A.

-4- (LU-factorization.) Compute the LU-factorization of

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 4 \\ -4 & -1 & -3 \end{bmatrix}$$

-5- (Inverse Matrix.) Compute the inverse of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

-6- (Cramer's Rule.) State and Prove Cramer's Rule.

-7- (Determinants.) For which value of t does

$$\det \begin{bmatrix} 1 & 2 & t \\ 3 & 4 & 5 \\ 6 & 7 & 0 \end{bmatrix} = 0?$$

-8- (True or False.) Mark the following statements as true or false by circling **F** or **T**, respectively. You need not give reasons for your answers.

1. T F To form the matrix product AB the matrix B has to have as many columns as A has rows.

2. T F If A and B are matrices that are not square, and the product AB can be formed, then the product BA cannot be formed.

3. T F Suppose A is a square matrix. It is invertible if and only if there is no right hand side $\mathbf{b}$ for which the linear system $A\mathbf{x} = \mathbf{b}$ has several solutions.

4. T F The square matrix A is invertible if and only if A^T is invertible.

5. T F Suppose A and B are both square matrices. Then if AB is invertible, so is B.

6. T F For two $n \times n$ matrices A and B, the determinant of their product equals the product of their determinants.

7. T F For two $n \times n$ matrices A and B, the determinant of their sum equals the sum of their determinants.

8. T F Suppose c is a scalar, and A is an $n \times n$ matrix. Then $\det(cA) = c \det A$.

9. T F Suppose T is a triangle formed by the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{u} + \mathbf{v}$ in $\mathbb{R}^2$. Then the area a of T is given by

$$a = |\det [\, \mathbf{u} \quad \mathbf{v} \,]| \,.$$

10. T F The inverse of an invertible matrix A equals

$$A^{-1} = \frac{1}{\det A} C$$

where C is the matrix of cofactors of A.