Write your name here:

Math 2270-1 — Fall 2019 — Exam 1

1	2	3	4	5	6	7	Total

Instructions

1. **This exam is closed books and notes. Do not use a calculator or other electronic devices. Do not use scratch paper.**
2. Use these sheets to record your work and your results. Use the space provided, and the back of these pages if necessary. **Show all work.** Unless it's obvious, indicate the problem each piece of work corresponds to, and for each problem indicate where to find the corresponding work.
4. To avoid distraction and disruption **I am unable to answer questions during the exam.** If you believe there is something wrong with a problem state so, and if you are right you will receive generous credit. I will also be unable to discuss individual problems and grading issues with you after you are done while the exam is still in progress.
5. If you are done before the allotted time is up I recommend strongly that you stay and use the remaining time to **check your answers.**
6. When you are done hand in your exam, pick up an answer sheet, and leave the room. Do not return to your seat.
7. All questions have equal weight.
8. Clearly indicate (for example, by circling or boxing) your final answers.

-1- (Linear Systems.) Find all solutions of the linear system

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

-2- (Row Echelon Form.) Compute the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

Indicate the pivot rows and columns.

-3- (Linear Transformations.) Let T be the linear transformation such that if $\mathbf{y} = T(\mathbf{x})$ then $\mathbf{y}$ is the vector obtained from $\mathbf{x}$ by rotating $\mathbf{x}$ **clockwise** by the angle t. Compute the standard matrix of the transformation.

-4- (Linear Transformations and Linear Systems.) Let

$$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

be the linear transformation satisfying

$$T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 3 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 8 \\ -2 \end{bmatrix}.$$

Compute the standard matrix A of T.

-5- (Linear Independence.) Consider the set of three vectors

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \right\}$$

Is this set linearly independent? Why or why not?

-6- (Row Echelon Forms.) List all possible row echelon forms of a 2×2 matrix. Use 0 to denote the entry zero, a bullet $\bullet$ to denote a pivot, and an asterisk $*$ or plus $+$ to denote an arbitrary entry.

-7- (True or False.) Mark the following statements as true or false by circling **F** or **T**, respectively. You need not give reasons for your answers.

1. T F A linear system of 2 equations in 3 unknowns always has a solution.

2. T F The linear system $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ is a linear combination of the columns of A.

3. T F A linear system of 3 equations in 2 unknowns cannot have a solution.

4. T F In a linearly dependent set of vectors every vector can be written as a linear combination of the other vectors.

5. T F In a linearly independent set of vectors no vector can be written as a linear combination of the other vectors.

6. T F For the linear system $A\mathbf{x} = \mathbf{b}$ to have a unique solution A must have at least as many rows as it has columns.

7. T F The span of the set of columns of a 3×2 matrix may be all of $\mathbb{R}^3$.

8. T F The standard matrix of a linear transformation from $\mathbb{R}^s$ to $\mathbb{R}^t$ is an $s \times t$ matrix.

9. T F The range (image) of a linear transformation contains the origin of its codomain.

10. T F A linear system can have a unique solution only if it is square.