Write your name here:

Math 2270-1 — Fall 2019 — November 6, 2019 — Exam 3

1	2	3	4	5	6	7	8	Total

Instructions

1. **This exam is closed books and notes. Do not use a calculator or other electronic devices. Do not use scratch paper.**
2. Use these sheets to record your work and your results. Use the space provided, and the back of these pages if necessary. **Show all work.** Unless it's obvious, indicate the problem each piece of work corresponds to, and for each problem indicate where to find the corresponding work.
4. To avoid distraction and disruption **I am unable to answer questions during the exam.** If you believe there is something wrong with a problem state so, and if you are right you will receive generous credit. I will also be unable to discuss individual problems and grading issues with you after you are done while the exam is still in progress.
5. If you are done before the allotted time is up I recommend strongly that you stay and use the remaining time to **check your answers.**
6. When you are done hand in your exam, pick up an answer sheet, and leave the room. Do not return to your seat.
7. All questions have equal weight.
8. Clearly indicate (for example, by circling or boxing) your final answers.

-1- (Coordinate Vectors.) Let V be the vector space of quadratic polynomials. Compute the coordinate vector $[p]_{\mathrm{B}}$ of the polynomial

$$p(x) = (2x-1)(x-3)$$

with respect to the basis

$$\mathrm{B} = \left\{1, x, x^2\right\}$$

-2- (Dimensions.) Recall that a (real) square matrix A is symmetric if

$$A = A^T.$$

Let V be the space of symmetric $n \times n$ matrices. What is the dimension of V if $n = 3$? What is it for general n?

-3- (Isomorphism.) Construct an isomorphism from the vector space of quadratic polynomials to the vector space of upper triangular 2×2 matrices.

-4- (Compute Eigenvalues and Eigenvectors.) Compute the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} -2 & 6 \\ 6 & 7 \end{bmatrix}$$

-5- (Gershgorin Theorem.) State and prove the Gershgorin Theorem.

-6- (Singularity versus Defectiveness.) Complete the following Table:

	singular	**invertible**
defective:		
diagonalizable:		

-7- (True or False.) Mark the following statements as true or false by circling **F** or **T**, respectively. You need not give reasons for your answers.

1. T F All finite dimensional spaces are isomorphic to $\mathbb{R}^n$ for some n.

2. T F The set of singular $n \times n$ matrices forms a linear space.

3. T F The set of antisymmetric square matrices (those satisfying $A = -A^T$) form a linear space.

4. T F Two isomorphic vector spaces have the same dimension.

5. T F The set of all sequences

$$y_0, y_1, y_2, \ldots$$

satisfying the infinitely many equations

$$y_{n+3} = y_{n+2} - 2y_{n+1} + y_n, \qquad n = 0, 1, 2, \ldots$$

form a liner space.

6. T F The set of all triangular matrices forms a linear space.

7. T F Suppose A is an invertible $n \times n$ matrix and V is a subspace of R^n of dimension k. Then the set

$$W = \{\mathbf{w} \in \mathbb{R}^n : \mathbf{w} = A\mathbf{v} \quad \text{for some } \mathbf{v} \in V\}$$

is a linear space with the same dimension as V.

8. T F Suppose V and W are subspaces of $\mathbb{R}^n$. Then the set of all vectors in $\mathbb{R}^n$ that can be written as $\mathbf{v} + \mathbf{w}$ where $\mathbf{v}$ is in V and $\mathbf{w}$ is in W, is a subspace of $\mathbb{R}^n$.

9. T F The union of two subspaces of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$.

10. T F The intersection of two subspaces of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$.

-8- (True or False.) Mark the following statements as true or false by circling **F** or **T**, respectively. You need not give reasons for your answers.

1. T F Every singular matrix is defective.

2. T F Row operations preserve eigenvalues but not eigenvectors.

3. T F A square matrix may not have any eigenvalues.

4. T F Similar matrices have the same eigenvalues.

5. T F Similarity transforms preserve the null space of a matrix.

6. T F All square matrices are similar to a diagonal matrix.

7. T F The eigenvalues of a real matrix may be complex.

8. T F The eigenvalues of a symmetric matrix may be complex.

9. T F Every defective matrix is singular.

10. T F A matrix is invertible if and only if all of its eigenvalues are non-zero.