Math 2270 10/1/19

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$$|A| = -\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 0 \end{vmatrix} = -\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = -4$$

$$= -(-2)\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = -4$$

$$-\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \qquad -\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \text{cofactors} \qquad \begin{bmatrix} 0 & -2 & 2 \\ 1 & 1 & -1 \\ & & \end{bmatrix}$$

$$+\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \qquad -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\begin{bmatrix} +\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} \\ -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \\ +\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 0 & -2 & 2 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} = C$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} 0 & 1 & -1 \\ -2 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} = \frac{1}{\det A} C^T$$

LU factorization

$$A = LU \qquad L = \begin{bmatrix} 1 & & 0 \\ x & \ddots & \\ x & x & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} x & & x \\ & \ddots & \\ 0 & & x \end{bmatrix}$$

$$Ax = L\underbrace{Ux}_{y} = b$$

$$Ly = b$$

$$Ux = y$$

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 1 & 2 \\ 6 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 1 \\ 2 & -3 & 0 \\ 3 & -6 & -1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2 & 2 & 1 \\ 2 & -3 & 0 \\ 3 & 2 & -1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 4 & 1 & 2 \\ 6 & 0 & 2 \end{bmatrix}$$

Cramer's Rule

$Ax = b$ $\quad$ A invertible

$|A| \neq 0$

$$x_i = \frac{|A_i|}{|A|}$$

$Ae_i = a_i$

$$A_i = [a_1, a_2, \ldots, a_{i-1}, \underset{\substack{\uparrow \\ \text{i-th column}}}{b}, a_{i+1}, \ldots, a_n]$$

$Ax = b$

$$I_i = [e_1, e_2, \ldots, e_{i-1}, x, e_{i+1}, \ldots, e_n]$$

$$AI_i = A_i$$

$$|A||I_i| = |A_i| = |A|\, x_i \qquad x_i = \frac{|A_i|}{|A|}$$

$$|I_i| = \left|\begin{bmatrix} 1 & & \vert & & 0 \\ & \ddots & \vert & & \\ & 1 & \vert & & \\ & & \vert & 1 & \\ 0 & & \vert & \ddots & \\ & & \underset{x}{\vert} & & 1 \end{bmatrix}\right| = x_i$$

$$\begin{bmatrix} x & & & x \\ 0 & \ddots & x & \\ \vdots & 0 & \ddots & \\ 0 & \cdots & 0 & x \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$x_1 = \frac{\left|\begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix}\right|}{\left|\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right|} = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{12} a_{21}}$$

$$x_2 = \frac{a_{11} b_2 - a_{21} b_1}{a_{11} a_{22} - a_{12} a_{21}}$$