Math 2270-1 9/10/19

Review

standard matrix of a linear transformation

$T: \mathbb{R}^n \to \mathbb{R}^m$ $y = T(x)$

$y \in \mathbb{R}^m$ $x \in \mathbb{R}^n$

T linear: $T(u+v) = T(u) + T(v)$

$T(cu) = cT(u)$

$T(x) = Ax$ A $m \times n$

$A = [a_1, \dots, a_n]$ $x = \begin{bmatrix} x_1 \\ | \\ x_n \end{bmatrix}$

$$Ax = \sum_{i=1}^{n} x_i a_i$$

$$a_i = T(e_i)$$

$e_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow$ i-th place

$e_i \in \mathbb{R}^n$

$$T(x) = T\left(\sum_{i=1}^{n} x_i e_i\right) \qquad x = \sum_{i=1}^{n} x_i e_i$$

$$= \sum_{i=1}^{n} T(x_i e_i)$$

$$= \sum_{i=1}^{n} x_i T(e_i)$$

$$= \sum_{i=1}^{n} x_i a_i$$

$$= Ax$$

$m = n = 2$

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$T: \mathbb{R}^2 \to \mathbb{R}^2$

$$T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}3\\4\end{bmatrix}$$

$$T(x) = Ax$$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}5\\6\end{bmatrix}$$

$$A = \begin{bmatrix}a & b\\c & d\end{bmatrix} \quad \begin{bmatrix}1\\2\end{bmatrix} \quad \begin{bmatrix}a+2b\\c+2d\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix} \quad \begin{bmatrix}1\\1\end{bmatrix} \quad \begin{bmatrix}a+b\\c+d\end{bmatrix} = \begin{bmatrix}5\\6\end{bmatrix}$$

$$A = \begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix} \qquad T(x) = Ax \quad x = \begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$$Ax = x_1\begin{bmatrix}1\\3\\5\end{bmatrix} + x_2\begin{bmatrix}2\\4\\6\end{bmatrix}$$

$$Ax = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{aligned} x_1 + 2x_2 &= 1\\ 3x_1 + 4x_2 &= 1\\ 5x_1 + 6x_2 &= 1 \end{aligned}$$

$x_2 = 1 \quad x_1 = -1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 5 & 6 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & -2 \\ 0 & -4 & -4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 2 = 1 \rightarrow x_1 = -1$

$-2x_2 = -2$

$x_2 = 1$

$$T(x) = Ax = b$$

A ref

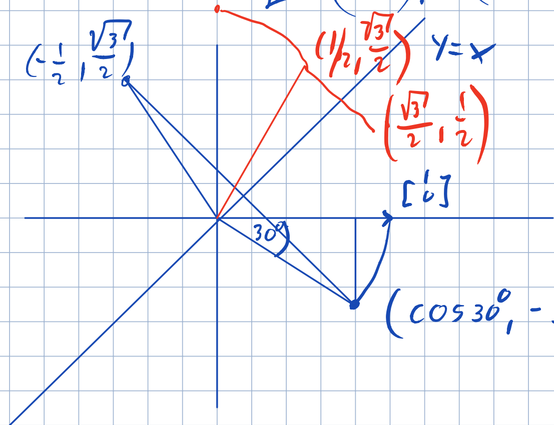
$$\begin{bmatrix} \bullet & \bullet & \\ & & \\ & & \circ \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} b_1 \\ | \\ b_m \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$T(x)$: rotate x clockwise by 30^0

and then reflect in $x=y$

$$A = \left[T\left(\begin{bmatrix}1\\0\end{bmatrix}\right), T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) \right] \qquad A = \begin{bmatrix} -1/2 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 1/2 \end{bmatrix}$$



$(\cos 30^0, -\sin 30^0) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Domain set of all inputs

Range (Image) set of all outputs

codomain $\mathbb{R}^n \supset$ Range

$A = [a_1, a_2, \dots, a_n]$

$T: \mathbb{R}^n \to \mathbb{R}^m$ linear $T(x) = Ax$

A $m \times n$

domain: $\mathbb{R}^n$

codomain: $\mathbb{R}^m$

range $= \text{span}\{a_1, \dots, a_n\}$

$S = \{ v_1, \dots, v_n \}$ linearly independent

if $\sum_{i=1}^{n} c_i v_i = 0 \Longrightarrow c_i = 0 \quad i = 1, \dots, n$

S linearly dependent

$\{v_1, v_2, v_3\}$ $3v_1 + 2v_2 \overset{+0v_3}{\downarrow} = 0$

$3v_1 + \dots = 0$

$v_1 = -\frac{2}{3} v_2$

$v_1 = \frac{1}{3} (- + \dots)$

$v_2 = -\frac{3}{2} v_1$

Ex.: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$Ax = b$

$Ay = b$

$y - x \neq 0$

$A(y - x) = 0$