Math 2270-1

Notes of 09/16/2019

Review of inverse matrices

- A square ($n \times n$) matrix A is said to be **invertible** if there exists an $n \times n$ matrix C such that
$$AC = CA = I$$
- We call C the **inverse of** A and denote it by A^{-1} (pronounced "A-inverse".
- If A is not invertible we say it is **singular** (or, less frequently, **non-invertible**).
- A non-square matrix ($m \times n$ with $m \neq n$) does not have an inverse. It is neither invertible, nor singular.
- We'll discuss properties of rectangular matrices that are resemble singularity or invertibility in the future.

- If A is invertible and $A\mathbf{x} = \mathbf{b}$ then $\mathbf{x} = A^{-1}\mathbf{b}$.

- If A is invertible and $AB = C$ then $B = A^{-1}C$.

- If A is invertible and $BA = C$ then $B = CA^{-1}$.

Assuming A and B are invertible and have the same size,

$$(AB)^{-1} = B^{-1}A^{-1}$$

In general

$$B^{-1}A^{-1} \neq A^{-1}B^{-1}$$

since matrix multiplication does not commute.

- In general, when you multiply with matrices it matters from which side you multiply. Left and right multiplying are different.
- Here is an example for a somewhat more complicated computation. Suppose all relevant matrices are invertible and have compatible sizes. Solve the matrix equation

$$(A - AX)^{-1} = X^{-1}B$$

for X.