Note: Math Center open on
Reading Day (Friday)
10:00am - 6:00pm

Wednesday 12/11 10:30 - 12:30 LCB 215
Q & A

Thursday 12/12 8:00 - 10:00am LCB 219
Final Exam

Gershgorin

If $\quad Ax = \lambda x \quad x \neq 0$

Then $\quad |a_{ii} - \lambda| \leq \sum_{j \neq i} |a_{ij}| \quad$ for some i

and $\quad |a_{kk} - \lambda| \leq \sum_{j \neq k} |a_{jk}| \quad$ for some k

why? $\quad$ Suppose $Ax = \lambda x$

$$\max_j |x_j| = 1 = x_i$$

$$\left(\begin{aligned} Ax &= \lambda x \qquad \Big| \cdot c \\ cAx &= c\lambda x \\ A(cx) &= \lambda(cx) \end{aligned} \right)$$

i-th equ of $Ax = \lambda x$

$$\sum_{j=1}^{n} a_{ij} x_j = \lambda x_i = \lambda \qquad \Big| - a_{ii} x_i$$

$$\Leftrightarrow$$

$$\lambda - a_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j$$

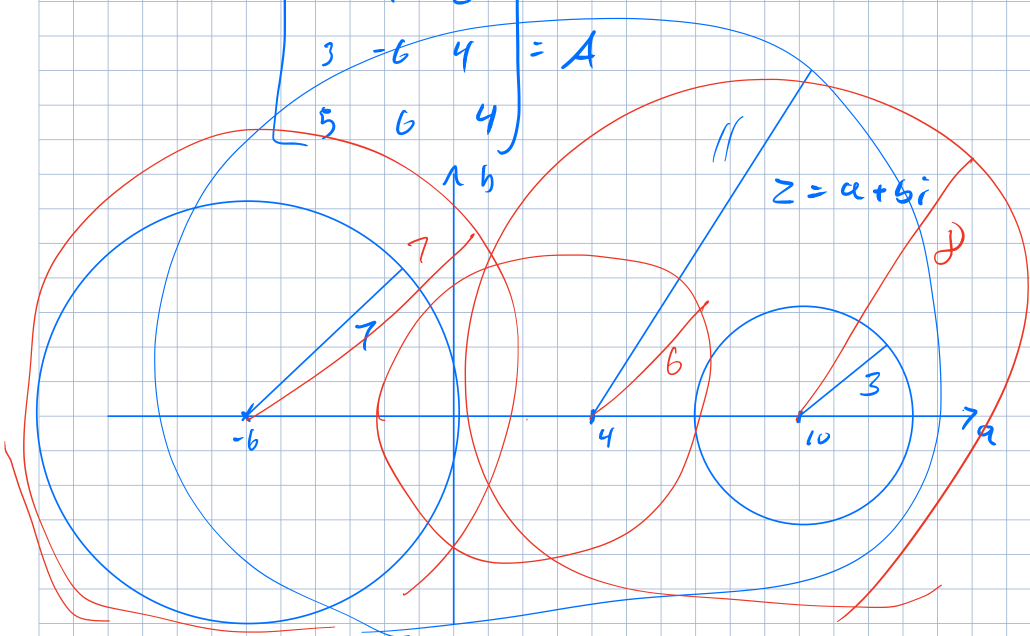
$$|\lambda - a_{ii}| = \Big| \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j \Big|$$

$$\leq \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| |x_j|$$

$$\leq \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$$

Ex.:

$$\begin{bmatrix} 10 & 1 & 2 \\ 3 & -6 & 4 \\ 5 & 6 & 4 \end{bmatrix} = A$$



eigenvalues are in

Union of row circles $\cap$ Union of Column Circles

If $\sum_{j \neq i} |a_{ij}| < |a_{ii}| \quad i = 1, \ldots, n$

$\Longrightarrow$ A is invertible

Positive Definiteness $\qquad A^T = A$

A positive definite if

$$x \neq 0 \Rightarrow x^T A x > 0$$

$$F: \mathbb{R}^n \to \mathbb{R}$$

Taylor Series: (about x_0) $\quad x_0 \in \mathbb{R}^n$

$$F(x) = F(x_0) + \nabla F(x_0)^T (x - x_0) + \frac{1}{2}(x - x_0)^T \nabla^2 F(x_0)(x - x_0)$$

$$\nabla F = \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{bmatrix}$$ gradient

$$\nabla^2 F = \left[\frac{\partial^2 F}{\partial x_i \partial x_j} \right] \quad i, j = 1, \ldots, n$$

Hessian

$A = A^T \quad a_{ii} > 0$ and A is diagonally dominant

$\Longrightarrow$ A is positive definite

Example

Least Squares

$$\|Ax-b\|^2 = \min$$

Normal Eqns

$$A^TAx = A^Tb$$

A $m \times n$ $\quad A^TA = n \times n$

Is A^TA p.d.

$$x^TA^TAx = y^Ty \quad y = Ax$$

$$\geq 0$$

A $m \times n$ $\quad m > n$

A^TA positive definite if rank $A = n$

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row ops

$$A \Leftrightarrow \underset{\text{ref.}}{R}$$

span(rows of R) $\subset$ span(rows of A)

" " " A C " " " R