

Math 2270-1

Matrix Multiplication

$$\begin{array}{ccc} & & B \quad n \times p \\ & & \begin{bmatrix} b_{11} & \dots & b_{1j} & \dots & b_{1p} \\ \vdots & & \vdots & & \vdots \\ b_{i1} & \dots & b_{ij} & \dots & b_{ip} \\ \vdots & & \vdots & & \vdots \\ b_{n1} & \dots & b_{nj} & \dots & b_{np} \end{bmatrix} \\ \begin{bmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{m1} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix} & \begin{bmatrix} & & & & \\ & & & & \\ & & \vdots & & \\ \dots & & c_{ij} & \dots & \\ & & \vdots & & \end{bmatrix} & \\ A \quad m \times n & & C = AB \quad m \times p \end{array}$$



It is evident from this picture that

- the $i - j$ entry of C is the (dot) product of the i -th row of A and the j -th column of B ,
- the j -th column of C is the product of A and the j -th column of B ,
- the i -th row of C is the product of the i -th row of A and B .

$$\begin{array}{c}
 B \quad n \times p \\
 \left[\begin{array}{ccccc}
 b_{11} & \dots & b_{1j} & \dots & b_{1p} \\
 \vdots & & \vdots & & \vdots \\
 b_{i1} & \dots & b_{ij} & \dots & b_{ip} \\
 \vdots & & \vdots & & \vdots \\
 b_{n1} & \dots & b_{nj} & \dots & b_{np}
 \end{array} \right] \\
 \\
 \left[\begin{array}{ccccc}
 a_{11} & \dots & a_{1j} & \dots & a_{1n} \\
 \vdots & & \vdots & & \vdots \\
 a_{i1} & \dots & a_{ij} & \dots & a_{in} \\
 \vdots & & \vdots & & \vdots \\
 a_{m1} & \dots & a_{mj} & \dots & a_{mn}
 \end{array} \right] \quad \left[\begin{array}{ccc}
 & \vdots & \\
 & & \vdots \\
 \dots & c_{ij} & \dots \\
 & \vdots & \\
 & & \vdots
 \end{array} \right] \\
 \\
 A \quad m \times n \qquad C = AB \quad m \times p
 \end{array}$$