Math 2270-1

Notes of 09/16/2019

Review of inverse matrices

- A square ($n \times n$) matrix A is said to be **invertible** if there exists an $n \times n$ matrix C such that

$$AC = CA = I$$

- We call C the **inverse of** A and denote it by A^{-1} (pronounced “A-inverse”).
- If A is not invertible we say it is **singular** (or, less frequently, **non-invertible**).
- A non-square matrix ($m \times n$ with $m \neq n$) does not have an inverse. It is neither invertible, nor singular.
- We'll discuss properties of rectangular matrices that are resemble singularity or invertibility in the future.

- If A is invertible and $A\mathbf{x} = \mathbf{b}$ then $\mathbf{x} = A^{-1}\mathbf{b}$.

- If A is invertible and $AB = C$ then $B = A^{-1}C$.

$$A^{-1}AB = A^{-1}C$$
$$B = A^{-1}C$$

- If A is invertible and $BA = C$ then $B = CA^{-1}$.

Assuming A and B are invertible and have the same size,

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(AB)(B^{-1}A^{-1}) = A\underbrace{BB^{-1}}_{I}A^{-1} = AA^{-1} = I$$

In general

$$B^{-1}A^{-1} \neq A^{-1}B^{-1}$$

since matrix multiplication does not commute.

- In general, when you multiply with matrices it matters from which side you multiply. Left and right multiplying are different.

- Here is an example for a somewhat more complicated computation. Suppose all relevant matrices are invertible and have compatible sizes. Solve the matrix equation

$$(A - AX)^{-1} = X^{-1}B \qquad \Big|\ X\cdot$$

for X. $\overset{?}{=}$

$$X(A-AX)^{-1} = XX^{-1}B = B \qquad \Big|\ \cdot(A-AX)$$

$$X = B(A-AX) = BA - BAX \qquad \Big|\ +BAX$$

$$X + BAX = BA$$

$$X(I+BA) \neq (I+BA)X = BA$$

$$X = (I+BA)^{-1}BA$$

$$B = \begin{bmatrix} 1 & 1 \\ 1 & -3 \\ -1 & 2 \end{bmatrix}$$

$$A \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = AB = I$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ -2 & -5 & -8 \\ 1 & 3 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 4 \\ 0 & -2 & -4 \\ 0 & 3/2 & 3 \end{bmatrix}$$

$$A + A^{-1} = 0$$

$$a + \frac{1}{a} = 0$$

$$a = -\frac{1}{a}$$

$$a^2 = -1$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$