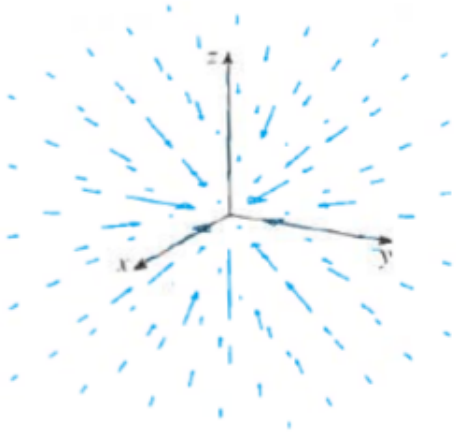
Math 1321 Week 11 Lab Worksheet Due Thursday 04/17

1. Any physical interpretation of a line integral $\int_C f(x,y,z)ds$ depends on the physical interpretation of the function f. Suppose that $\rho(x,y,z)$ represents the linear density at a point (x,y,z) of a thin wire shaped like a curve C. Then the mass, m, of the wire may be obtained by $\int_C \rho(x,y,z)\ ds$.

   (2 points) Find the mass of a spring in the shape of the helix defined parametrically by $x = 2\cos(t)$, $y = t$, $z = 2\sin(t)$ for $0 \leq t \leq 6\pi$, with density $\rho(x,y,z) = 2y$.

Many forces can be modeled with *conservative* force fields. Gravitational fields can be modeled this way. This fact allows us to estimate the total work done by a force field on an object passing through the field.



2. **(2 points)** Suppose that $\mathbf{F}$ is an inverse square force field, that is

$$\mathbf{F}(\mathbf{r}) = \frac{c\mathbf{r}}{|\mathbf{r}|^3}$$

for some constant c and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Determine a formula for the work done by moving an object from a point P_1 to the point P_2 in terms of the distances d_1 and d_2 of these points from the origin. *Hint: If* $\mathbf{F}$ *is conservative, then line integrals are independent of the path.*

3. **(1 point)** The gravitational force field is modeled by

$$\mathbf{F}(\mathbf{r}) = \frac{-mMG\mathbf{r}}{|\mathbf{r}|^3}$$

Using your answer to part (1), determine the work done by the gravitational field when the earth moves from *aphelion* to *perihelion*. These correspond to the earth's maximum distance of 1.52×10^8 km from the sun, and it's minimum distance of 1.47×10^8 km from the sun respectively. Use the values of $m = 5.97 \times 10^{24}$ kg, $M = 1.99 \times 10^{30}$ kg, and $G = 6.67 \times 10^{-11}$ $\mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$.

4. **(1 point)** Determine the volume of the solid bounded by the cone $z^2 = x^2 + y^2$ and the paraboloid $z = x^2 + y^2$.

5. **extra practice (1 make-up point)** Determine the volume of the solid bounded by the circular cylinder $(x - a)^2 + y^2 = a^2$, the cone $z = x^2 + y^2$ and the plane $z = 0$.(Hint: Use cylindrical coordinates).