Name: Score:

Math 1321 Week 10 Lab Due Thursday 4/10

1. **Warm-up (2 points)** Sketch the region of integration and interchange the order and evaluate.

$$\int_0^1 \int_{1-y}^1 (x + y^2) dx dy$$

2. **Polar Coordinates**

(a) **(2 points)** Evaluate $\int\int_{D_a} e^{-(x^2+y^2)}dxdy$ where D_a is the disk $x^2 + y^2 \leq a$.

(b) **(1 points)** Show that the limit as $a \to \infty$ is π.

3. **Triple Integrals (2 points)** Let W be the region bounded by the planes $x = 0$, $y = 0$, and $z = 2$, and the surface $z = x^2 + y^2$. Compute $\int\int\int_W x dx dy dz$.

4. **(1 point for effort + 1 make-up point for correct solution)** There is no direct way to compute the following integral using x, y coordinates.

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \pi$$

Yes, believe it or not, the answer is π! In fact, prove it. Hint: Use your answers to the antepenultimate question and *don't* use polar coordinates this time.

(By the way, the $\int_{-\infty}^{\infty} e^{-x^2} dx$ is called the Gaussian Integral and it plays a role in modern probability theory and quantum mechanics.)