Name: Score:

Math 1321 Week 8 Lab Worksheet Due Thursday 3/20

1. **Gradients**– The temperature (in degrees Celsius) near Dawson Creek at noon on April 15, 1901 is given by $T = -(0.0003)x^2y + (0.9307)y$, where x and y are the latitude and longitude(in degrees). (The latitude and longitude of Dawson Creek are $x = 55.7°$ and $y = 120.2°$)

   (a) (**1 point**) At what rate is the temperature changing if we proceed directly north?

   (b) (**1 point**) Near Dawson Creek, in what direction is the rate of change in temperature the greatest. Is the temperature increasing or decreasing in that direction? Write you answer as a cardinal direction, i.e. north or northwest.

   (c) (**1 point**) Where is Dawson Creek Geographically?

2. (**Level Curves and Harmonics**) A function $u = f(x, y)$ with continuous second derivatives satisfying *Laplace's equation*

$$u_{xx} + u_{yy} = 0$$

is called a *harmonic function.*

(a) (**1 point**) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic.

(b) (**1 point**) Is $u(x, y) = e^x \sin y$ harmonic?

(c) (**1 point**) Suppose $u = x^2 - y^2$ and $v = xy$. Show that the level curves of u are perpendicular to the level curves v.

(d) (**1 point**) Show that u and v are harmonic.

(e) (**Make-up 1 point**) Suppose that u = f(x,y) and $v = g(x, y)$ have continuous partial derivatives which satisfy the *Cauch-Riemann equations*: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$. Show that the level curves of u are perpendicular to v

3. **Chain Rule and Partial Derivatives**– The advection equation is a simple model derived from the conservation of mass describing the transport of a signal or wave carried along by a stream with fixed velocity. If we denote the concentration of the chemical by $u(x,t)$, then the advection equation has the form

$$u_t = -cu_x$$

. The constant c is called the propagation speed of the stream.

(a) (**1 point**) Use partial differentiation to show that $u(x,t) = e^{-(x-t)^2}$ is a solution to the advection equation.

(b) (**1 point**) Use the chain rule to show that if $g(x)$ is the initial distribution of the concentration of the pollutant, the $g(x-ct)$ is a solution to the advection equation

(c) (**1 point**) Try to use the principle of conservation of mass to justify the use of $u_t = -cu_x$ as a model for advection(transport).

4. **Make-up question** The differential equation $u_t + uu_x + u_{xxx}$ is called the *Korteweg-de Vries Equation* (KdV). The KdV equation describes advection in the presence of "steepening" and dispersion of waves in *shallow* water, and thus serves as a more sophisticated model describing the motion of traveling waves along a channel called *solitons*.

(a) (**1 point**) Show that for $c > 0$, the function

$$u(x,t) = 3c\,\mathrm{sech}^2\left[\frac{\sqrt{c}}{2}(x - ct)\right]$$

is a solution to the *Korteweg-de Vries Equation*.

(b) (**1 point**) Compare the term uu_x with the term cu_x associated with the advection equation. Hint: recall that c is the propagation speed.

(c) (**1 point**) How do the speed and shape of the *soliton* depend on the size of the parameter c? Attach several plots to illustrate your solution.