Name: Score:

Math 1321 Week 2 Worksheet Due Thursday 01/23

1. **(10 points) Test for convergence or divergence. Cite any tests or theorems used in your solutions.**

   (a) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n^2+5}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

   (c) $\sum_{n=1}^{\infty} \frac{n!}{3^{n-1}}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^{n-3}\sqrt{n}}{n+4}$

(e) $\sum_{n=2}^{\infty} \frac{cos(n\pi)}{\sqrt{n}}$

2. Bond energy between salt ions Na^+ and Cl^-. Positive and negative charges form an attractive force f_a proportional to the inverse square of the distance r between the charges, which can cause the sodium and chloride to combine in a solid crystal

$$f_a(r) = -\frac{A}{r^2},$$

where A is a positive constant and the negative sign $-$ indicates that the force f_a pulls the two opposing charges together. For large distances r, the force is weak, but nearby the force gets stronger, much like the attractive force of a magnet. The two atoms also have a repulsive force that prevents the atoms from fusing infinitesimally close to each other.

$$f_b(r) = \frac{B}{r^n},$$

where B is a positive constant and the force goes with the inverse nth power of distance (the power is typically anywhere between the 7th and 13th power, depending on the particular atoms involved).

The net force f_{net} on the two ions is

$$f_{net}(r) = f_b(r) + f_a(r)$$

The ions are in a stationary equilibrium when their position r^* is such that the net force is zero: $f_{net} = 0$.

(a) **(2 points)**Solve for r^*

(b) **(2 points)**From the equilibrium point, compute the bonding energy of the two molecules by computing the work involved to move the two ions from r^*, to a position infinitely far apart. That is, compute the work done against the force $f_{net}(r)$ as we pull apart the ions by making r go to infinity to verify that $W(r^*) = \frac{C}{r^*}$ where C is a positive constant that depends on n and A.