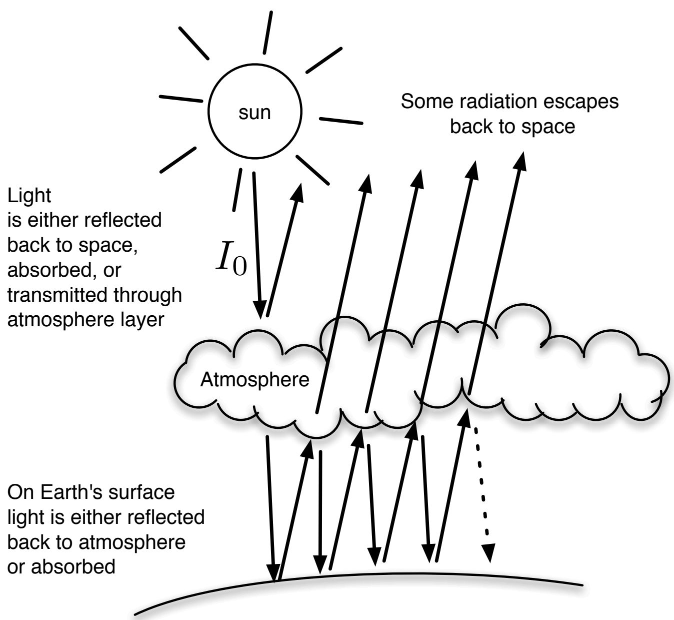
Name: Score:

Math 1321 Worksheet 1 Due Thursday 01/16/14

1. **(1 point)** Determine if the sequence $a_n = \frac{n^3}{3n^3+1}$ converges as $n \to \infty$. Does the series $\sum_{n=1}^{\infty} a_n$ converge?

2. **(2 points) Bouncing ball:** Suppose a ball is dropped from a height of 2m and begins to bounce. The height of the second bounce is $\frac{3}{2}$m, while the height of the third bounce is $\frac{9}{8}$m, and so on indefinitely. What is the total vertical distance traveled by the ball?

3. **(5 points) Bouncing Sunbeams**: A sun ray with intensity I_0 is directed at the Earth (see below Figure). A fraction R_a of the ray's intensity is reflected off the atmospheric layer(depicted as a cloud) back into space. Another fraction A_a is absorbed in the atmosphere, and the remaining fraction T_a is transmitted through the atmosphere to the Earth's surface. These three fractions account for all the incoming light, therefore $R_a + T_a + A_a = 1$. The fraction transmitted through the atmosphere undergoes a similar fractionation on the Earth's surface, with a fraction R_e being reflected back, and the remaining fraction $A_e = 1 - R_e$ being absorbed (no radiation transmits through the earth—too thick). The fraction of radiation R_e reflected back upward into the atmosphere undergoes a further reflection-transmission-absorption fractionation, with some radiation (T_a) escaping to outer space, the remainder either being absorbed as heat or re-reflected back to earth in an infinite cycle on the increasingly small fraction of remaining radiation intensity. As you will see, this back-and-forth process in our atmosphere can cause a greater amount of light to be absorbed compared to a planet without an atmosphere.



(a) **(2 points) Assume $T_a = 0.4$, $R_a = 0.3$, and $R_e = 0.5$. Compute the fraction of the original intensity $I_0 = 1$ that escapes back to space. Hint: Add up all the transmitted-to-space fractions and use your knowledge of series to calculate the infinite sum. Don't plug in numerical values until the end.**

(b) **(1 point) With parameters as in (a), what is the fraction absorbed by the earth?**

(c) **(2 points) If the atmosphere did not exist, then $T_a = 1$. What intensity will be absorbed by the planet? Compare to (b). This is a simple model of the "greenhouse" effect. CO_2 increases the absorptive fraction A_a of the Earth's atmosphere, causing greater temperature on earth.**