Handout 24

Recall: Fundamental Theorem of Calculus $\int_a^b F'(x)dx = F(b) - F(a)$**.**

Definition: A vector field **F** is called a **conservative vector field** if there is exist a potential, a function *f*, s.t. $\mathbf{F} = \nabla f$.

Theorem: Let C be a smooth curve given by the vector function $\vec{r}(t), a \le t \le b$. Let **F** be a continuous conservative vector field, and f is a differentiable function (of 2/3 variables) that satisfy the equation $\mathbf{F} = \nabla f$. Then

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

Definition: Let vector field **F** be continuous on domain *D*. If $\oint_{C_1} \mathbf{F} \cdot d\mathbf{r} = \oint_{C_2} \mathbf{F} \cdot d\mathbf{r}$ for any two curves, C_1, C_2, with the same initial and end points, then the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ is called independent of path.

Corollary: A line integral of conservative vector field is independent of path\curve.

Definition: A curve C is called closed if its terminal points coincide.

Theorem: $\oint_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path in *D* if and only if $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any closed curve *C* in *D*.

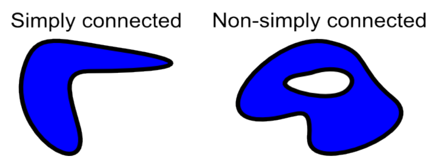
Theorem: Suppose **F** is a vector field that is continuous (all components are continuous) on an open connected region *D*. If $\oint_C \mathbf{F} \cdot d\mathbf{r}$ be independent of path in *D*, then **F** is conservative vector field on *D*, that is there is exists function *f* such that $\mathbf{F} = \nabla f$.

Theorem: If $\mathbf{F} = \langle P, Q \rangle$ is a conservative vector field where *P*,*Q* have continuous first order partial derivatives on domain *D* then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in *D*.

Proof: Since **F** is conservative there is differentiable function *f* such that $\mathbf{F} = \nabla f$ and therefore $f_{xy} = \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = f_{yx}$.

Definitions:
1) A **simply connected curve** is a curve that doesn't intersect itself between endpoints.
2) A **simple closed curve** is a curve with $\vec{r}(a)=\vec{r}(b)$ but $\vec{r}(t_1)\neq\vec{r}(t_2)$ for any $a<t_1<t_2<b$.
3) A **simply connected region**: is a region *D* in which every simple closed curve encloses only points from *D*. In other words *D* consist of one piece and has no hole.

simple curves *nonsimple curves*



Theorem: Let $\mathbf{F}=\langle P,Q\rangle$ be a vector field on an open simply connected region *D*. If P,Q have continuous first order partial derivatives on domain *D* and $\dfrac{\partial P}{\partial y}=\dfrac{\partial Q}{\partial x}$, then **F** is conservative.