Handout 20

Definition: For the purpose of integration we recognize 2 types of domains:

- Domain of type I is given by $D_I = \{(x,y) \mid a \le x \le b, h_1(x) \le y \le h_2(x)\}$
- Domain of type II is given by $D_{II} = \{(x,y) \mid \tilde{h}_1(y) \le x \le \tilde{h}_2(y), c \le y \le d\}$

Suppose the functions *f*, *g* are continues on the domains D_I, D_{II} respectively, then

$$\iint_{D_I} f(x,y)\,dA = \int_a^b \int_{h_1(x)}^{h_2(x)} f(x,y)\,dy\,dx \quad \text{and} \quad \iint_{D_{II}} g(x,y)\,dA = \int_c^d \int_{\tilde{h}_1(x)}^{\tilde{h}_2(x)} g(x,y)\,dx\,dy$$

Properties of Double Integrals:

1) $\iint_D f(x,y) + g(x,y)\,dA = \iint_D f(x,y)\,dA + \iint_D g(x,y)\,dA$

2) $\iint_D cf(x,y)\,dA = c\iint_D f(x,y)\,dA$

3) $f(x,y) \ge g(x,y) \Rightarrow \iint_D f(x,y)\,dA \ge \iint_D g(x,y)\,dA$

4) $D = D_1 \cup D_2 \Rightarrow \iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)\,dA$

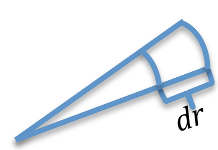
5) $\iint_D f(x,y)\,dA = A(D)$, i.e. the area of the domain D

6) $m \le f(x,y) \le M \Rightarrow mA(D) \le \iint_D f(x,y)\,dA \le MA(D)$

Note: When change variables\coordinates of integration from Cartesian to Polar coordinates the implied change in differentials is given by $dxdy = rdrd\theta$

Theorem: If *f* is continuous on a polar "rectangle" *R* given by $0 \le a \le r \le b, \alpha \le \theta \le \beta$ where $0 \le \beta - \alpha \le 2\pi$, then

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$



Theorem: If *f* is continuous of polar region of the form $D = \{(r,\theta) \mid \alpha \le \theta \le \beta, h_1(\theta) \le r \le h_2(\theta)\}$ then

$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$