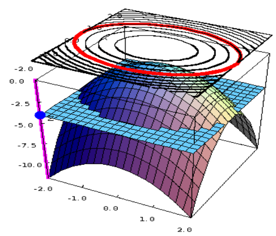
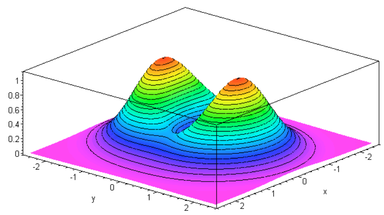
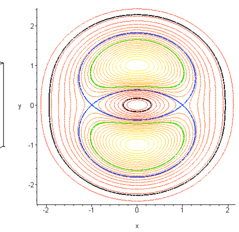
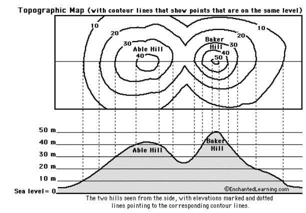
Handout 15

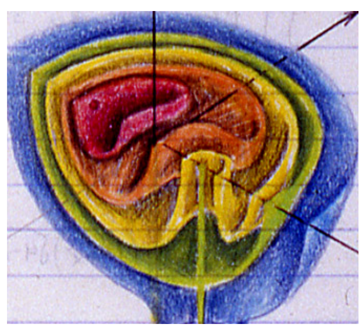
Definition: The **level curves\contour lines** of a function *f* of 2 variables are the curves with equations *f*(*x*,*y*)=*k,* where *k* is a constant in the range of *f*.
One can draw a surface in 2D using level curves as projections of the curves onto xy plane, often by using equally spaced set of constants *k*. The well-known example is the topographical maps.

Note: If *f*(*x*,*y*) is continuous (not defined yet), then the level curves at different heights are never cross.

Definition: The level surfaces of a function *f* of 3 variables are the surfaces with equations *f*(*x*,*y*,*z*)=*k* where *k* is a constant in the range of *f*.

One can use level surfaces to visualize functions of 3 variables, but there is no way to describe the real image, because the image is in 4th dimensions.



Definition: A function multiple variables can be viewed in several ways, each one may be useful in different situations; we may meet it in future.

1. A function of n real variables $f(x_1, x_2, \ldots, x_n)$
2. A vector function $f(\vec{x}) = f(\langle x_1, x_2, \ldots, x_n \rangle)$, where $\vec{x} = \langle x_1, x_2, \ldots, x_n \rangle$
3. A single point variable function $f((x_1, x_2, \ldots, x_n)) = f(x_1, x_2, \ldots, x_n)$, where the n-tuple $(x_1, x_2, \ldots, x_n)$ is considered a point on n-dimensional space.

Definition: We write $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and say the limit of f(x,y) as (x,y) approaches (a,b) is L if we can make the values of f(x,y) as close to L as we like by taking the point (x,y) sufficiently colose to the point (a,b), but not equal to (a,b).

Theorem: If the limit $\lim_{(x,y)\to(a,b)} f(x,y) = L$ exists then for any curve $(x,y) = (x(t), y(t))$ the function $\tilde{f}(t) = f(x(t), y(t))$ has the limit at t_0 s.t. $(a,b) = (x(t_0), y(t_0))$ and $\lim_{t\to t_0} \tilde{f}(t) = L$.

Note: If we find 2 different curves $(x(t), y(t))$ and $(\tilde{x}(t), \tilde{y}(t))$ for which $\lim_{(x(t),y(t))\to(a,b)} f(x(t), y(t)) = L_1$ and $\lim_{(\tilde{x}(t),\tilde{y}(t))\to(a,b)} f(\tilde{x}(t), \tilde{y}(t)) = L_2$, such that $L_1 \neq L_2$ then the limit $\lim_{(x,y)\to(a,b)} f(x,y)$ doesn't exists.

Definition: A function *f* of two variables is called **continues at point (a,b)** if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.

Definition: A function *f* of two variables is called **continues** if *f* is continuous at every $(a,b) \in D$, where *D* is the domain of *f*.