# Handout 12

Reminder: **y =f(x)** mean that a function **f** uses a variable (an ingredient) **x** to make the result **y**.

Definition: two variable function **z=f(x,y)** uses variables\ingredients x, y to make the result **z**.
A domain of function of two variables are ordered pair (x,y) for which f is defined and the image is the set of values of f(x,y).

Examples of functions of two variables:
a. Elevation of earth is a function of earth coordinates
b. Volume of circular cylinder is function of radius and height. $V = \pi r^2 h$
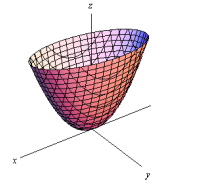c. z=x+y – is a a plane x+y-z=0
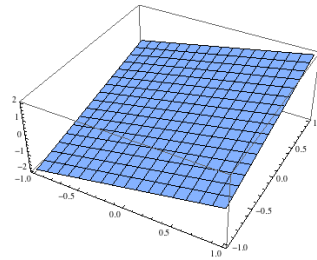
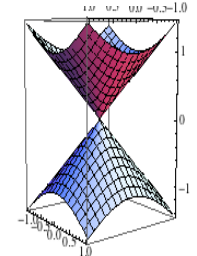**Definition**: Quadratic surfaces are function of the form
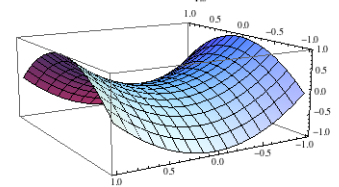$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G$$
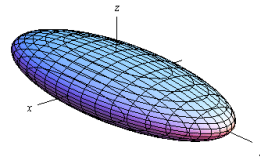
Examples

a. Elliptic paraboloid $z = x^2 + y^2$.

b. Cone $z^2 = x^2 + y^2$: fixing z give circle at either positive and negative height (symmetrical), fixing x=0 gives $z = \pm y$
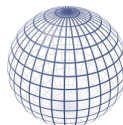

c. Hyperbolic parabaloid (saddle) $z^2 = y^2 - x^2$

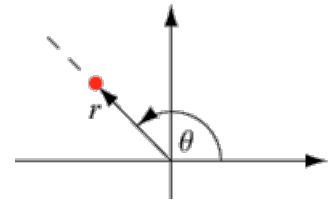d. Ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$
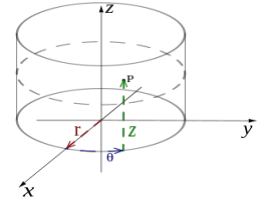

e. Sphere $x^2 + y^2 + z^2 = r^2$

Polar Coordinate system:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Leftrightarrow \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \end{cases} \quad r \geq 0, \quad 0 \leq \theta < 2\pi$$

Cylindrical Coordinates:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \Leftrightarrow \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \\ z = z \end{cases} \quad r \geq 0, \quad 0 \leq \theta < 2\pi$$

Spherical Coordinates:

$$\begin{cases} x = \rho\sin\varphi\cos\theta \\ y = \rho\sin\varphi\sin\theta \\ z = \rho\cos\varphi \end{cases} \Leftrightarrow \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\theta = \dfrac{y}{x} \\ \cos\varphi = \dfrac{z}{\rho} \end{cases} \quad r \geq 0, 0 \leq \theta < 2\pi, 0 \leq \varphi \leq \pi$$