Handout 11

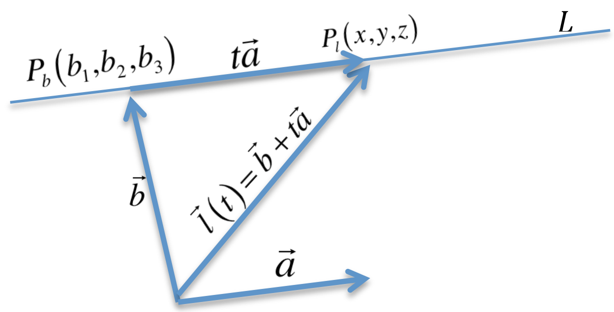
Recall: parametric curves $(x,y)=(f(t),g(t)),\quad a\le t\le b$ describe the curve by describing each point in the xy plane

Similarly in 3D: $(x,y,z)=(f(t),g(t),h(t)),\quad a\le t\le b$
Similarly in vector form: $\vec{r}(t)=\langle f(t),g(t),h(t)\rangle$
Equations of line:

Functional form: $y=ax+b=\dfrac{A}{\tilde{A}}x+\dfrac{B-\tilde{B}}{\tilde{A}}$

Parametric form: $(x,y)=(\tilde{A}t+\tilde{B},At+B)$; $(x,y,z)=(a_1t+b_1,a_2t+b_2,a_3t+b_3)$

Vector form: $\vec{l}(t)=\langle a_1t+b_1,a_2t+b_2,a_3t+b_3\rangle=t\langle a_1,a_2,a_3\rangle+\langle b_1,b_2,b_3\rangle=\vec{a}t+\vec{b}$



Yet another way to express line in 3D is by

$$(x,y,z)=(x_0+at,y_0+bt,z_0+ct)\Rightarrow\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}(=t)$$

Parallel lines: The lines $\vec{l}_1(t)=\vec{a}t+\vec{b}$, $\vec{l}_2(t)=\vec{c}t+\vec{d}$ are parallel iff $\vec{c}=k\vec{a}$.
Intersecting lines: The lines $\vec{l}_1(t)=\vec{a}t+\vec{b}$, $\vec{l}_2(t)=\vec{c}t+\vec{d}$ if there is t_0 such that $\vec{l}_1(t_0)=\vec{l}_2(t_0)$

Equation of Plane:

- $Ax+By+Cz=D$
- $\langle A,B,C\rangle\cdot(\langle x,y,z\rangle-\langle x_0,y_0,z_0\rangle)=0$ or
- $\langle A,B,C\rangle\cdot\langle x,y,z\rangle=\langle A,B,C\rangle\cdot\langle x_0,y_0,z_0\rangle=D$
- $\vec{n}=\langle A,B,C\rangle,x\doteq\langle x,y,z\rangle,x_0=\langle x_0,y_0,z_0\rangle$ gives $\vec{n}(\vec{x}-\vec{x}_0)=0$ or $\vec{n}\vec{x}=\vec{n}\vec{x}_0$.

Theorem: The angle between planes is the angle between their normal vectors.

Definition: The distance between a point $P_1(x_1, y_1, z_1)$ and a plane $n_1x + n_2y + n_3z + d = 0$ is Is given by $D = \frac{|n_1x_1 + n_2y_1 + n_3z_1 + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Notes:

- To find distance between parallel planes choose a point on one and use previous formula.
- To find distance between skew lines find the distance between their planes.