Handout 9

Definition: The dot product is an operation between two vectors that results in a scalar, in a component form is given by:

$$\vec{a} = \langle a_1, a_2 \rangle, \vec{b} = \langle b_1, b_2 \rangle: \qquad \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

$$\vec{c} = \langle c_1, c_2, c_3 \rangle, \vec{d} = \langle d_1, d_2, d_3 \rangle: \quad \vec{c} \cdot \vec{d} = c_1 d_1 + c_2 d_2 + c_3 d_3$$

Definition: The geometric definition of the dot product between 2 vectors **u,v** (in either 2D or 3D) is given by $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\alpha$, where α is the angle between the vectors **u** and **v.**

Definition: An angle between 2 vectors defined $\alpha = \arccos\left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$

Definition: Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal iff $\vec{a} \cdot \vec{b} = 0$

Properties of the Dot Product:

1) $\mathbf{a} \cdot \mathbf{a} = a_1^2 + a_2^2 + a_3^2 = |\mathbf{a}|^2$

2) $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = b_1 a_1 + b_2 a_2 + b_3 a_3 = \mathbf{b} \cdot \mathbf{a}$

3) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \langle b_1 + c_1, b_2 + c_2, b_3 + c_3 \rangle = \langle a_1(b_1 + c_1), a_2(b_2 + c_2), a_3(b_3 + c_3) \rangle =$

$= \langle a_1 b_1 + a_1 c_1, a_2 b_2 + a_2 c_2, a_3 b_3 + a_3 c_3 \rangle = \langle a_1 b_1, a_2 b_2, a_3 b_3 \rangle + \langle a_1 c_1, a_2 c_2, a_3 c_3 \rangle = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

4) $(c\,\mathbf{a}) \cdot \mathbf{b} = (ca_1)b_1 + (ca_2)b_2 + (ca_3)b_3 = (ca_1)b_1 + (ca_2)b_2 + (ca_3)b_3 =$

$= c(a_1 b_1) + c(a_2 b_2) + c(a_3 b_3) = c(\mathbf{a} \cdot \mathbf{b})$

$= a_1(cb_1) + a_2(cb_2) + (ca_3)b_3 = \mathbf{a} \cdot (c\,\mathbf{b})$

5) $0 \cdot \mathbf{a} = 0$

Definition:

A **Scalar Projection** of vector **a** on vector **b** (also called a signed magnitude or a component of the projection) is denoted as $\text{comp}_{\mathbf{b}}\,\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

A **Vector Projection:** of vector **a** on vector **b** $\text{proj}_{\mathbf{b}}\,\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|} \frac{\mathbf{b}}{|\mathbf{b}|} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2} \mathbf{b}$

Note: The common mistake is to mix up the vectors, therefore