Course: Accelerated Engineering Calculus II	Instructor: Michael Medvinsky

Handout 2

Definition: A sum of infinite sequence $\{a_n\}_{n=1}^{\infty}$, $S=\sum_{j=1}^{\infty} a_j$ is called **(infinite) series**.

Definition: In order to find a sum of infinite sequence $S=\sum_{j=1}^{\infty} a_j$ one defines a sequence of **partial sums** as $\{S_n\}_{n=1}^{\infty}=\left\{\sum_{j=1}^{n} a_j\right\}_{n=1}^{\infty}$. If the sequence is convergent then $S=\lim_{n\to\infty} S_n$ and the series is called **convergent**. Otherwise the series is **divergent**.

Theorem: If $S=\sum_{n=1}^{\infty} a_n$ is convergent series, then

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}(S_{n+1}-S_n)=\lim_{n\to\infty} S_{n+1}-\lim_{n\to\infty} S_n=S-S=0$$

Note: The converse theorem doesn't true.

The test for Divergence: If $\lim_{n\to\infty} a_n \neq 0$ or DNE then $S=\sum_{n=1}^{\infty} a_n$ is divergent.

Theorem: If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent series, then so are the series $\sum_{n=1}^{\infty} ca_n$ (c is constant) and $\sum_{n=1}^{\infty}(a_n \pm b_n)$. Furthermore: $\sum_{n=1}^{\infty} ca_n = c\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty}(a_n \pm b_n)=\sum_{n=1}^{\infty} a_n \pm \sum_{n=1}^{\infty} b_n$.

The Integral test: Let f be continues, positive decreasing function on $[m,\infty)$ and let $a_n=f(n)$ for $n\geq m$ then the series $\sum_{n=m}^{\infty} a_n$ converges if and only if $\int_m^{\infty} f(x)dx$.

Note: Since finite number of terms cannot affect convergence of infinite series, it is enough that $f(x)$ decreasing on $[M,\infty)$ where $M>m$.

The Comparison Test: Let $\sum a_n, \sum b_n$ be series with $a_n, b_n > 0$, then

- If $\sum b_n$ is convergent and $a_n \le b_n$ then $\sum a_n$ is also convergent
- If $\sum b_n$ is divergent and $a_n \ge b_n$ then $\sum a_n$ is also divergent

The Limit Comparison Test: Let $\sum a_n, \sum b_n$ be series with $a_n, b_n > 0$. If $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = c$ where $c > 0$ is a finite constant, then either both series converge or both diverge.

Definition: Let $a_n > 0$, then

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 - + \ldots$$
$$\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty} (-1)^{n-1} a_n = a_1 - a_2 + a_3 - a_4 + - \ldots$$

called alternating series.

Theorem: If an alternating series, either

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} (-1)^n a_n = -a_1 + a_2 - a_3 + a_4 - + \ldots \text{ or}$$

$$\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty} (-1)^{n-1} a_n = a_1 - a_2 + a_3 - a_4 + - \ldots,$$

where $a_n > 0$ satisfy 1) $b_{n+1} \le b_n$ and 2) $\lim\limits_{n\to\infty} b_n = 0$. Then the series are converges.