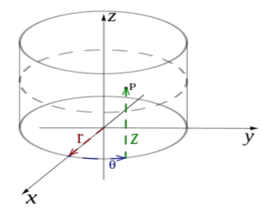
2.7 Cylindrical and Spherical Coordinates (9.7)

In 2D we often use Polar coordinate system (r,θ) in addition to Cartesian (x,y) ones (actually more coordinate system exists). Some time one is more convenient then another. We want to expand the idea of polar coordinates to 3D.

First, **recall** the Polar Coordinate system:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Leftrightarrow \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \end{cases} \qquad r \ge 0, \quad 0 \le \theta < 2\pi$$

There is 2 different ways to expand the Polar Coordinates to 3D, the Cylindrical Coordinates and Spherical Coordinates.



2.7.1 Cylindrical Coordinates

The easiest ones is the Cylindrical coordinates, which are given

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \Leftrightarrow \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \\ z = z \end{cases} \qquad r \ge 0, \quad 0 \le \theta < 2\pi$$

Ex 1. Determine surface of $z^2 = r^2$: a cone

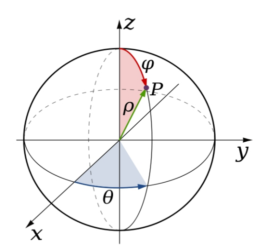
Ex 2. Write hyperboloid $x^2 + y^2 - z^2 = 2$ in cylindrical coordinates: $r^2 - z^2 = 2$

Ex 3. Convert $(x,y,z) = (1,2,3)$ to cyl. coords: $r = \sqrt{1^2 + 2^2} = \sqrt{5}, \theta = \arctan 2, z = 3$

Ex 4. Convert $(r,\theta,z) = (1,2,3)$ to Cartesian coordinates $x = 1\cdot\cos 2, y = 1\cdot\sin 2, z = 3$

2.7.2 Spherical Coordinates

$$\begin{cases} x = \rho\sin\varphi\cos\theta \\ y = \rho\sin\varphi\sin\theta \\ z = \rho\cos\varphi \end{cases} \Leftrightarrow \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\theta = \dfrac{y}{x} \\ \cos\varphi = \dfrac{z}{\rho} \end{cases} \qquad r \ge 0, 0 \le \theta < 2\pi, 0 \le \varphi \le \pi$$



Ex 5. Determine surface of $\cos\varphi = \rho$: this elliptic paraboloid $z = \rho\cos\varphi = \rho^2$

Ex 6. Write hyperboloid $x^2 + y^2 - z^2 = 2$ in cylindrical coordinates: $r^2 - z^2 = 2$

Ex 7. Convert $(x,y,z) = (1,2,3)$ to Sph coords $\rho = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}, \theta = \arctan 2, \varphi = \arccos 3/\sqrt{14}$

Ex 8. Convert $(\rho,\theta,\varphi) = (1,2,3)$ to Cart coords $x = 1\cdot\sin 3\cos 2,\ y = 1\cdot\sin 3\sin 2,\ z = 1\cdot\cos 3$