

2.6 Functions and Surfaces (9.6)

Reminder: $y=f(x)$ mean that a function f uses a variable (an ingredient) x to make the result y .

Def: two variable function $z=f(x,y)$ uses variables\ingredients x, y to make the result z .

A domain of function of two variables are ordered pair (x,y) for which f is defined and the image is the set of values of $f(x,y)$.

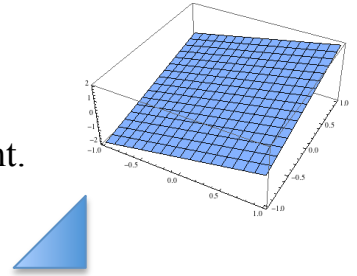
Examples of functions of two variables:

Ex 1. Elevation of earth is a function of earth coordinates

Ex 2. Volume of circular cylinder is function of radius and height.

Ex 3. $z=x+y$ – is a a plane $x+y-z=0$

Ex 4. Find domain of $\frac{x+y}{\sqrt{x-y}}$: the domain is $x-y > 0 \Rightarrow x > y$

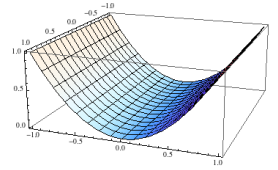


2.6.1 Quadratic surfaces

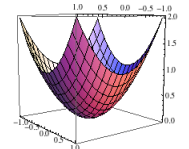
Def: Quadratic surfaces are function of the form

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G$$

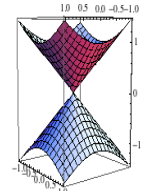
Ex 5. $f(x,y) = y^2$



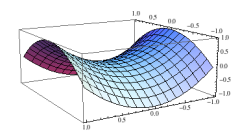
Ex 6. Elliptic paraboloid $z = x^2 + y^2$: fixing z gives circles at positive height, fixing $x=0$ give parabola.



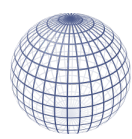
Ex 7. Cone $z^2 = x^2 + y^2$: fixing z give circle at either positive and negative height (symmetrical), fixing $x=0$ gives $z = \pm y$



Ex 8. Hyperbolic paraboloid (saddle) $z^2 = y^2 - x^2$:
 $x=0 : z = y^2, y=0 : z = -x^2$



Ex 9. Sphere $x^2 + y^2 + z^2 = 0$



Ex 10. Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$

