4 Logarithmic functions (1.6)

If $1 \neq a > 0$, the function a^x is either strictly increasing or strictly decreasing. This is definitely one-to-one function and is onto $\mathbb{R}^+$, therefore the inverse function is exists. Furthermore, it is a Logarithmic function: $\log_a x = y \Leftrightarrow a^y = x$

Ex 1. $\log_{10} 0.001 = -3$ and $10^{-3} = 0.001$

A property of inverse function gives: $\begin{matrix} \log_a a^x = x, \forall x \in \mathbb{R} \\ a^{\log_a x} = x, \forall x > 0 \end{matrix}$ particularly $\log_a a = 1$.

Laws of logarithm:

$$\log_a(xy) = \log_a x + \log_a y$$
$$\log_a(x/y) = \log_a x - \log_a y$$
$$\log_a x^r = r\log_a x, \forall r \in \mathbb{R}$$

Change of base: $\log_a x = \dfrac{\log_b x}{\log_b a}$

Common bases: 2, *e*, 10, they often denoted as $\log_2 = \lg$ (binary logarithm, often used in computer sciences) $\log_{10} = \mathrm{Log}$ (decadic\decimal logarithm, last time I met it was in my school) , $\log_e = \ln$ (natural logarithm, most convenient form for math and for this course).

Ex 2. Compute following logs

Solution:

$$\log_2 32 = \log_2 2^5 = 5$$
$$\log_2 \frac{1}{16} = \log_2 1 - \log_2 16 = 0 - 4 = -4$$
$$\log_{32} 16 = \frac{\log_2 16}{\log_2 32} = \frac{\log_2 2^4}{\log_2 2^5} = \frac{4}{5}$$

Ex 3. Solve the equation $\ln(3+x) + \ln(1+x) = 3\ln 2$

$$\ln(3+x)(1+x) = \ln 2^3 \Rightarrow (3+x)(1+x) = 8$$
$$3 + 4x + x^2 = 8 \Rightarrow x^2 + 4x - 5 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{16+20}}{2} = \frac{-4 \pm 6}{2} = 1, \cancel{-5}$$

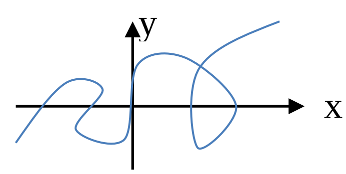
Note that the solution of quadratic equation, -5, will lead to log of negative number which is undefined. Therefore -5 don’t solve the original equation.

Verification: $\ln(3-5)+\ln(1-5)=\cancel{\ln(-2)+\ln(-4)}$
$\ln(3+1)+\ln(1+1)=\ln 4+\ln 2=\ln 8=3\ln 2$

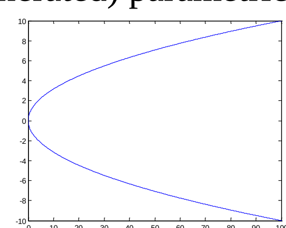
Ex 4. Solve equation
$$3e^{2x}-8e^{x}+16=0$$
$$4e^{2x}-e^{2x}-8e^{x}+16=0$$
$$\left(2e^{x}\right)^{2}-2\cdot 4\cdot e^{x}+4^{2}-e^{2x}=\left(2e^{x}-4\right)^{2}-e^{2x}=0$$
$$2e^{x}-4=\pm e^{x}\Rightarrow 2e^{x}\pm e^{x}=4\Rightarrow\begin{cases}3e^{x}=4\\ e^{x}=4\end{cases}\Rightarrow\begin{cases}x=\ln\frac{4}{3}\\ x=\ln 4\end{cases}$$

5 Parametric Curves (1.7)

Consider you observe a flying insect and want to describe the trajectory of it's flight. It is impossible to describe that curve by an equation of the form y=f(x), because it isn't function (see example).



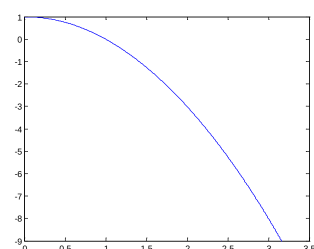
Parameterization of curves is the solution to the problem above. A parametric curve is defined by pair of functions x=f(t) and y=g(t) where **t** is the **parameter**. One can think about regular curve that represent a function as a (degenerated) parametric curve with f(t)=t.



Ex 5. Write a parametric form $x=y^{2}$ and sketch it.

Solution: $x=t^{2}, y=t$

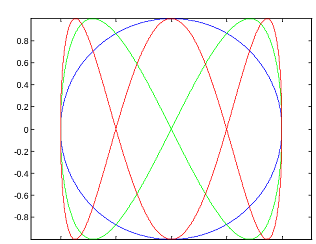
Ex 6. Sketch $x=\sqrt{t}, y=1-t$



Solution:

t	0	1	4	9
x	0	1	2	3
y	1	0	-3	-8

Note that if we substitute $t=x^{2}$ we get parabola $y=1-x^{2}$, however the parameterization doesn't defined on the same domain as the parabola.



Ex 7. Sketch $x=\cos t, y=\sin at$ for a=1,2,3