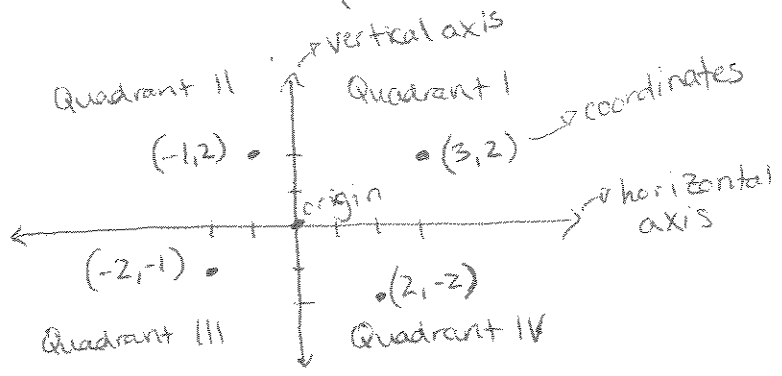
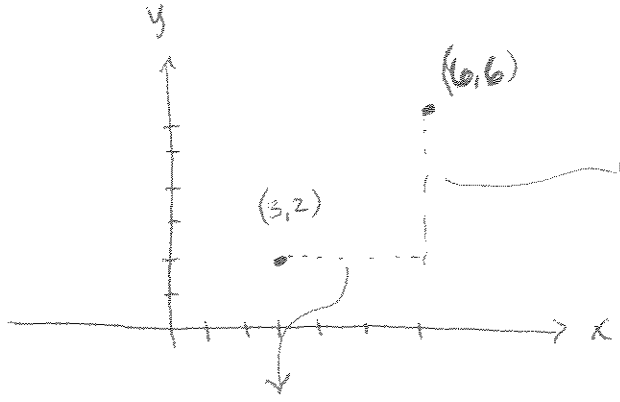
3.1 - Rectangular Coordinate System

→ also called Cartesian plane



Distance



vertical distance: 4

$(6-2)$ or $|6-2|$

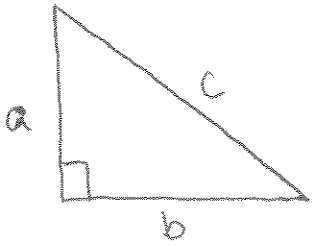
in general $|y_2 - y_1|$

horizontal distance: 3

$(6-3)$ or $|6-3|$ → don't have to worry about which number is to the left on real number line.

in general $|x_2 - x_1|$

Suppose we have a right triangle:



Pythagorean Theorem

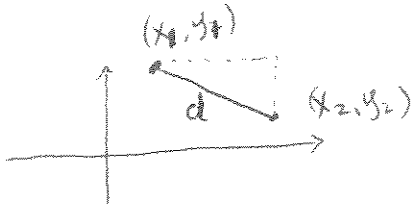
$a^2 + b^2 = c^2 \Rightarrow c = \sqrt{a^2 + b^2}$

Using the Pythagorean Thm the distance between $(3,2)$ and $(6,6)$ is

$$d = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

In general

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Notice, from our example

$$d = \sqrt{(6-3)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = 5$$

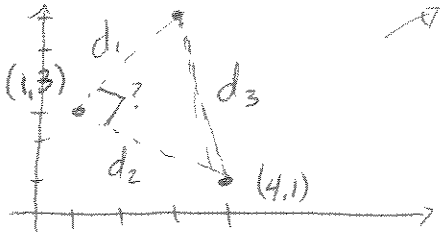
and

$$d = \sqrt{(3-6)^2 + (2-6)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16} = 5$$

one way you order your points doesn't matter as long as you're consistent.

Ex Are the points $(1,3)$, $(4,1)$, and $(3,6)$ the vertices of a right triangle?

plot $(3,6)$



note: drawing not to scale

Is $d_1^2 + d_2^2 = d_3^2$? If so, it is a right triangle.

$$d_1 = \sqrt{(3-1)^2 + (6-3)^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$d_2 = \sqrt{(4-1)^2 + (1-3)^2} = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$d_3 = \sqrt{(4-3)^2 + (1-6)^2} = \sqrt{1^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$$

EX cont...

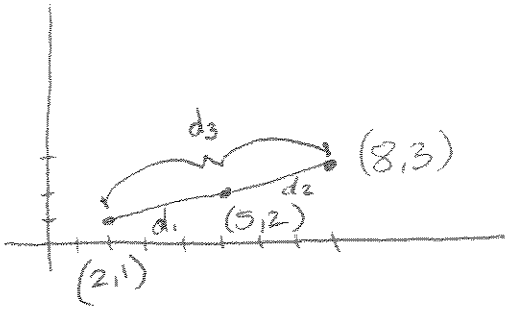
Notice $d_1^2 + d_2^2 = \left(\sqrt{13}\right)^2 + \left(\sqrt{13}\right)^2 = 13 + 13 = 26$

and $d_3^2 = \left(\sqrt{26}\right)^2 = 26$

so the points are on the vertices of a right triangle.

EX Are the points (8,3), (5,2), and (2,1) collinear? (all lie on same line)

Use distance formula.



Collinear if $d_1 + d_2 = d_3$

$d_1 = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$

$d_2 = \sqrt{(8-5)^2 + (3-2)^2} = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$

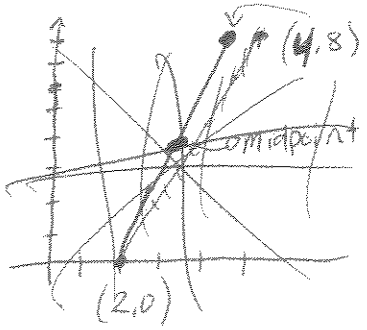
$d_3 = \sqrt{(8-2)^2 + (3-1)^2} = \sqrt{6^2+2^2} = \sqrt{36+4} = \sqrt{40}$

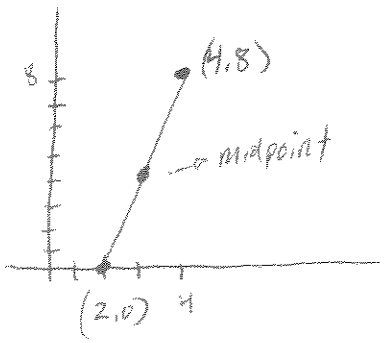
Now $d_1 + d_2 = \sqrt{10} + \sqrt{10} = 2\sqrt{10}$

$d_3 = \sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

yes, the points are collinear.

Suppose you plot the points (2,0) and (4,8) & connect them with a line. What is the point halfway between the two points? (called the midpoint)





X-value should be halfway between the two endpoint X-values:

$$\frac{2+4}{2} = 3$$

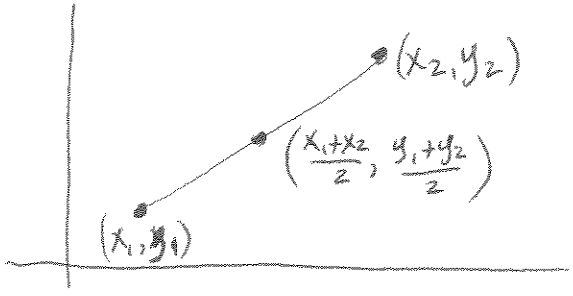
y-value halfway between endpoint y-values:

$$\frac{0+8}{2} = 4$$

so the midpoint is $(3,4)$

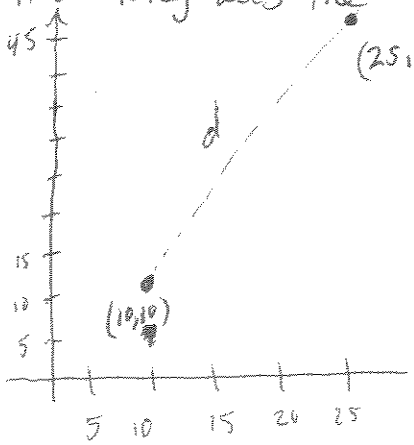
In general the midpoint is given by

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



EX Football:

QB throws pass from 10 yard line, 10 yards from sideline. Wide receiver catches ball on 46 yard line, 25 yards from same sideline. How long was the pass?



$$d = \sqrt{(25-10)^2 + (46-10)^2} = \sqrt{15^2 + 36^2}$$

$$= \sqrt{225 + 1296}$$

$$= \sqrt{1521}$$

$$= 39 \text{ yards}$$