# 2.5 - Absolute value Equations and Inequalities



→ Recall that the absolute value of a number (or expression) is its distance from zero.

→ Think about the equation

$$|x| = 2$$

→ There are two options:

$x = 2$ or $x = -2$

→ There are always 2 options in absolute value equations with the exception $|x| = 0$. Solution $x = 0$

Ex Suppose $|x-5| = 2$

$x - 5 = 2$ or $x - 5 = -2$

$x = 7$ $\qquad$ $x = 3$

Check: $|7-5| = |2| = 2$ ✓ $\qquad$ $|3-5| = |-2| = 2$ ✓

Ex $|2x - 1| = 3$

$2x - 1 = 3$ or $2x - 1 = -3$

$2x = 4$ $\qquad$ $2x = -2$

$x = 2$ $\qquad$ $x = -1$

Check: $|2 \cdot 2 - 1| = |4-1| = |3| = 3$ ✓ $\qquad$ $|2(-1) - 1| = |-2-1| = |-3| = 3$ ✓

Ex $|2x-1|+3=8$ $\quad \to -3$

$$|2x-1|=5$$

$2x-1=5$ or $2x-1=-5$

$2x=6$ $\quad$ $2x=-4$

$x=3$ $\quad$ $x=-2$

heck: $|2\cdot 3-1|+3=5+3=8$ ✓ $\quad |2(-2)-1|+3=|-5|+3=5+3=8$ ✓

→ Sometimes there is no solution

Ex: $|3x-2|=-5$ $\quad \to \boxed{\text{no solution}}$

Ex: $|5-2x|+10=6$ $\quad \to -10$

$|5-2x|=-4$ $\quad \boxed{\text{no solution}}$

Trickier Example

Ex $3|2x-5|+4=7$ $\quad \to -4$

$3|2x-5|=3$ $\quad \to \div 3$

$|2x-5|=1$

$2x-5=1$ or $2x-5=-1$

$2x=6$ $\quad$ $2x=4$

$x=3$ $\quad$ $x=2$

heck: $3|2\cdot 3-5|+4=3|6-5|+4=3|1|+4=3+4=7$ ✓

$3|2\cdot 2-5|+4=3|4-5|+4=3|-1|+4=3+4=7$ ✓

Ex $|4x-10| = 2|2x+3|$

$4x - 10 = 2(2x+3)$ or $4x - 10 = -2(2x+3)$

$4x - 10 = 4x + 6$ $\qquad$ $4x - 10 = -4x - 6$

$8x = 4$

$-10 \neq 6$ $\qquad$ $x = \frac{1}{2}$

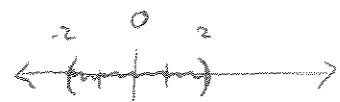
no solution

Check: $|4(\tfrac{1}{2}) - 10| = |2-10| = |-8| = 8 \rightarrow LHS$

$2|2(\tfrac{1}{2})+3| = 2|1+3| = 2|4| = 8 \rightarrow RHS$

## Inequalities

Suppose $|x| < 2$. On real number line




So $|x|<2$ really means $-2 < x < 2$

So then $|x-1| < 2$ means $-2 < x-1 < 2$

or $-1 < x < 3$

Ex $\left|\frac{2x-4}{5}\right| - 9 \leq 3 \Rightarrow \left|\frac{2x-4}{5}\right| \leq 12$

so $-12 \leq \frac{2x-4}{5} \leq 12 \rightarrow \cdot(5)$

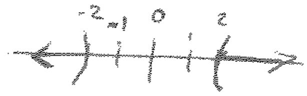
$-60 \leq 2x - 4 \leq 60 \rightarrow +4$

$-56 \leq 2x \leq 64 \rightarrow \div 2$

$-28 \leq x \leq 32$

$x \in [-28, 32]$

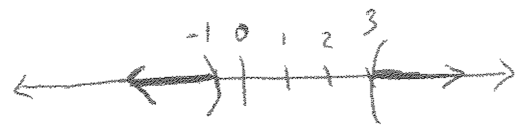
Now suppose $|x|>2$. The distance between $x$ and zero is greater than 2.




so $x>2$ or $x<-2$

Similarly $|x-1|>2$ means $x-1>2$ or $x-1<-2$

$x>3$ or $x<-1$




Ex $|2x+3|\geq 9$

$2x+3\geq 9$ or $2x+3\leq -9$

$2x\geq 6$ $\qquad 2x\leq -12$

$x\geq 3$ $\qquad x\leq -6$