2.2 Linear Equations and Problem Solving

→ In this section, we practice setting up and solving word problems.

Ex 1) Alice makes $10,000 more than three times as much as Bob. She makes $100,000. How much does Bob make?

- denote Bob's salary x.

$$100000 = 3x + 10{,}000 \qquad \to -10{,}000$$

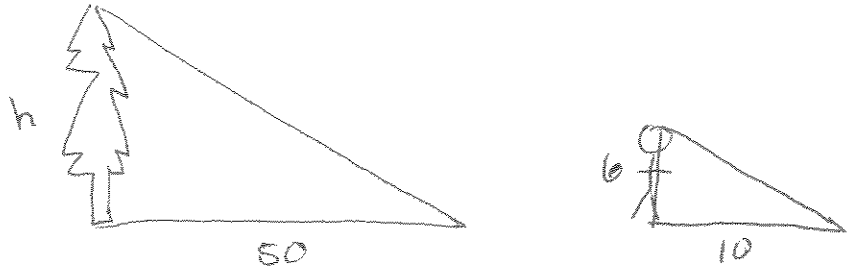
$$90{,}000 = 3x \qquad \to \div 3$$

$$30{,}000 = x$$

$$x = \$30{,}000$$

Ex 2) A tree casts a shadow of 50'. You are 6' tall and cast a shadow of 10'. How tall is the tree?

→ Draw a picture



- These are similar triangles.
- That means the ratio of height to length of shadow is the same for each triangle.

So,

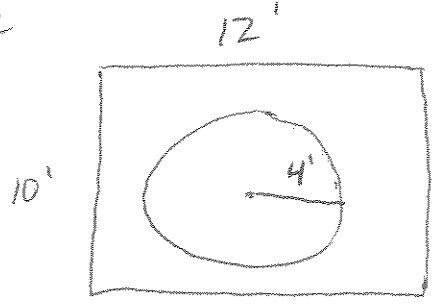
$$\frac{h}{50} = \frac{6}{10} \quad \rightarrow \cdot 50$$

$$h = \frac{6}{10_1} \cdot 50^{5}$$

$$h = 30 \text{ feet}$$

Ex: 3) The floor of a rectangular room that is 10' by 12' is partially covered by a circular rug with a radius of 4 feet. What percent of the floor is covered by the rug?

- Picture



- Area of floor: $10' \cdot 12' = 120 \text{ ft}^2$ → area of rectangle = length · width
- Area of rug: $\pi 4^2 = 16\pi \text{ ft}^2$ → area of circle = πr^2
- % covered by rug: $\frac{16\pi}{120_{6}} \cdot 100^{5} = \frac{80\pi^{40}}{6_{3}} = \frac{40\pi}{3} \approx 41.89\%$

Ex 4: You go out to dinner and your meal costs $18.75. (including tax) You want to leave a tip of 20%. How much do you pay overall?

$$\frac{20}{100} \cdot 18.75 = 0.2 \cdot 18.75$$

$$= 3.75 \rightarrow \text{tip}$$

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 18.75 \\ \times\ \ 0.2 \\ \hline 3.750 \end{array}$$

Total: $\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 18.75 \\ +\ 3.75 \\ \hline 22.50 \end{array}$ you pay $22.50

Ex 5 You can dig a hole in 5 hours. Your big brother can dig the same hole in 3 hours. How fast can you dig the hole together? (assume you can both work simultaneously).

- Your rate: $\frac{1}{5}$ hole/hour
- Bro's rate: $\frac{1}{3}$ hole/hour

- here, rate · time = amount of hole you've dug.
- let amount of time you both dig be t.
- you dig one hole, so

$$\frac{1}{5}t + \frac{1}{3}t = 1$$

$$\frac{3}{15}t + \frac{5}{15}t = 1$$

$$\frac{8}{15}t = 1 \qquad \rightarrow \cdot \frac{15}{8}$$

$$t = \frac{15}{8} \text{ hours}$$

$$\approx 1.875 \text{ hours}$$

Ex 6 A force of 32 lb. stretches a spring 6 inches. How much force is needed to stretch it 1.25 ~~feet~~ feet?

→ We need consistent units!

.25 feet = 3 inches

→ 1.25 feet = 15 inches

→ There is a constant ratio

$$\frac{32 \text{ lb}}{6 \text{ in}} = \frac{X \text{ lb}}{15 \text{ in}} \quad \rightarrow .(15)$$

$$\frac{32}{6} \cdot 15 = x$$

$$80 = x$$

80 lb of force

Ex 7 Which is a better price for ranch dressing?

32 oz for $4.99

12 oz for $2.99

→ Big: $\frac{\$4.99}{32 \text{ oz}} = \frac{\$0.156}{\text{oz}}$

Small: $\frac{\$2.99}{12 \text{ oz}} = \frac{\$0.249}{\text{oz}}$

Big bottle is a better price.