

2.1 Linear Equations

→ A linear equation is one that can be written

$$ax + b = 0$$

↳ The variable is raised to the first power

→ To solve, we want to isolate the variable by doing the same operation on both sides of the equation.

Ex: $4x - 12 = 0$ → add 12
 $4x = 12$ → divide by 4
 $x = 3$

→ often (but not always) do addition/subtraction before multiplication & division.

check: $4 \cdot 3 - 12 = 0$ ✓

Ex: $3x + 2 = 11$ → -2
 $3x = 9$ → ÷ 3
 $x = 3$

check: $3 \cdot 3 + 2 = 11$ ✓

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Ex: $x+2 = 2x-6$ $\rightarrow -2-2x$
 $-x = -8$ $\rightarrow \cdot(-1)$
 $x = 8$

check: $8+2=10$
 $2 \cdot 8-6=10$ ✓

Ex: $6(y+2) = 4y+6$ \rightarrow distribute
 $6y+12 = 4y+6$ $\rightarrow -4y-12$
 $2y = -6$ $\rightarrow \div 2$
 $y = -3$

check: $6(-3+2) = 6(-1) = -6$ ✓
 $4(-3)+6 = -12+6 = -6$ ✓

Ex: $\frac{t}{6} + \frac{t}{8} = 1$ \rightarrow multiply by lcm (24)
 $4t + 3t = 24$ \rightarrow combine like terms
 $7t = 24$ $\rightarrow \div 7$
 $t = \frac{24}{7}$

check: $\left(\frac{24}{7}\right) \cdot \frac{1}{6} + \left(\frac{24}{7}\right) \cdot \frac{1}{8} = \frac{24}{7} \cdot \frac{1}{6} + \frac{24}{7} \cdot \frac{1}{8} = \frac{4}{7} + \frac{3}{7} = \frac{7}{7} = 1$ ✓

EX: $2x - 4 = 2(x - 3)$ \rightarrow distribute
 $2x - 4 = 2x - 6$ $\rightarrow -2x + 4$
 $0 = -2$

\rightarrow False! This means there is no solution to the equation.

EX: $3x + 2 + 2(x - 6) = 5(x - 2)$ \rightarrow distribute
 $3x + 2 + 2x - 12 = 5x - 10$ \rightarrow combine like terms
 $5x - 10 = 5x - 10$

\rightarrow This statement is true for every x value. There are infinitely many solutions to the equation.

3 cases:

- 1) One solution
- 2) no solution
- 3) Infinitely many solutions

\rightarrow A linear equation can't have 2 solutions!

EX: $\frac{1}{x-2} = \frac{1}{2x+1}$ \rightarrow take reciprocal ^{of} ~~both~~ both sides
 $x - 2 = 2x + 1$ $\rightarrow -2x + 2$
 $-x = 3$ $\rightarrow \cdot (-1)$
 $x = -3$

check $\frac{1}{-3-2} = -\frac{1}{5}$; $\frac{1}{2(-3)+1} = \frac{1}{-6+1} = -\frac{1}{5}$ ✓

EX: $\frac{1}{x-2} = \frac{3}{2x+1}$

→ reciprocal

$x-2 = \frac{2x+1}{3}$

→ • 3

$3(x-2) = 2x+1$

→ distribute

$3x-6 = 2x+1$

→ $-2x+6$

$x=7$

check: $\frac{1}{7-2} = \frac{1}{5}$

$\frac{3}{2(7)+1} = \frac{3}{15} = \frac{1}{5}$

Aside: $(x+4)(x-1) = x(x-1) + 4(x-1)$
 $= x^2 - x + 4x - 4$
 First Outer Inner Last

EX: $\frac{x-1}{x-2} = \frac{x-3}{x+4}$

→ • $(x-2) \cdot (x+4)$

$(x+4)(x-1) = (x-3)(x-2)$

→ distribute FOIL

$x^2 - x + 4x - 4 = x^2 - 2x - 3x + 6$

→ combine like terms

$x^2 + 3x - 4 = x^2 - 5x + 6$

→ $-x^2 + 5x + 4$

~~8~~ $x = 10$

→ ~~8~~

$x = 10/8$

$x = 5/4$

~~check~~

check: $\frac{\frac{5}{4}-1}{\frac{5}{4}-2} = \frac{\frac{5}{4}-\frac{4}{4}}{\frac{5}{4}-\frac{8}{4}} = \frac{\frac{1}{4}}{-\frac{3}{4}} = \frac{1}{4} \cdot \left(-\frac{4}{3}\right) = -\frac{1}{3}$

$\frac{\frac{5}{4}-3}{\frac{5}{4}+4} = \frac{\frac{5}{4}-\frac{12}{4}}{\frac{5}{4}+\frac{16}{4}} = \frac{-\frac{7}{4}}{\frac{21}{4}} = -\frac{7}{4} \cdot \frac{4}{21} = -\frac{1}{3}$